# Tutorial: 3D toroidal physics - testing the boundaries of symmetry breaking

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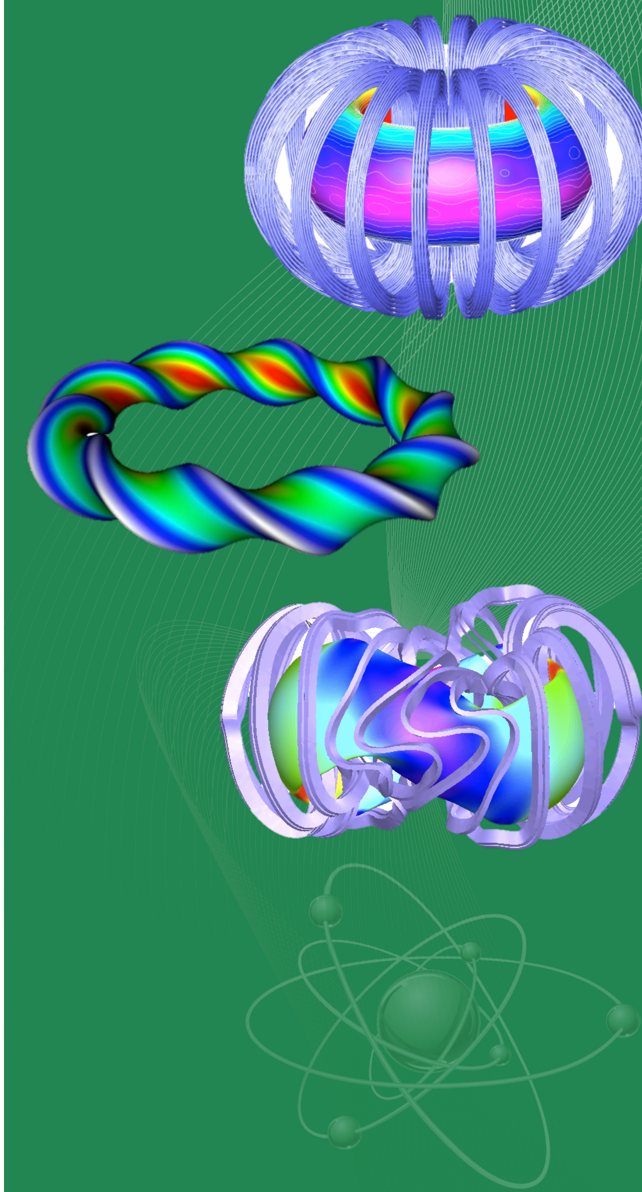
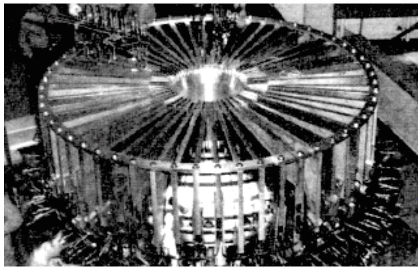


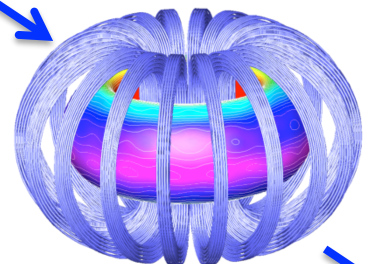
# 3D symmetry-breaking effects are present in all toroidal fusion configurations

- **Engineering/economic constraints**
  - Finite number of TF coils, ferrous steel structures (blankets, beams, etc.), error fields from fabrication tolerances
  - Particle/energy sources not symmetrically distributed (pellets,beams, RF)
- **Plasma generated**
  - Evolution to lower energy state: macro scale 3D instability structures
  - Single-helicity states in reversed field pinches (RFP)
  - Halo currents, pellet ablation clouds, neutrals, edge plasma blobs
- **Plasma control**
  - Coils for edge localized instabilities, resistive wall instabilities
- **Plasma optimization**
  - Vacuum rotational transform, confinement/stability optimization

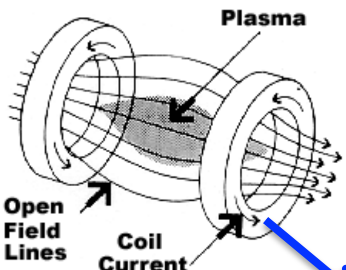
# The progression of 3D/symmetry-breaking in fusion: tokamaks, mirrors



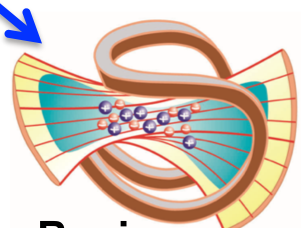
ORMAK (56 TF coils)



current devices
18-22 TF coils
up-down asymmetry

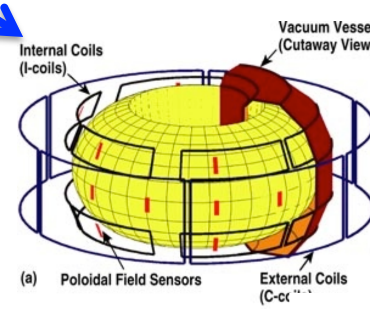



simple mirror

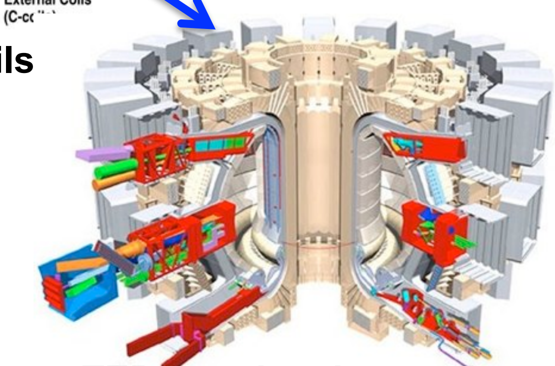


min B mirror

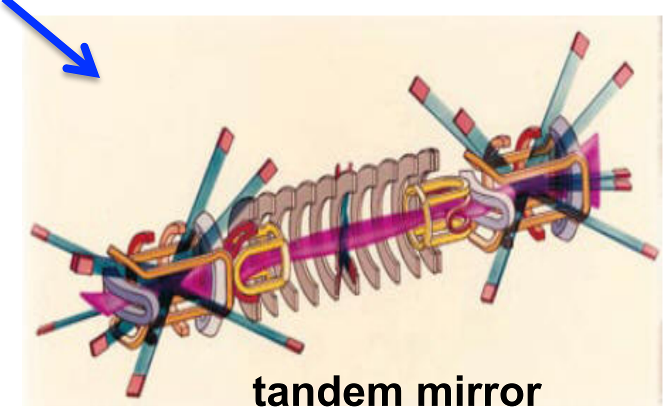
- Tokamaks initially 3D only from
  - Finite number of toroidal coils
  - Conducting shell breaks, field errors
- More recently
  - Fewer TF coils
  - ELM/RWM stability control coils
  - Non-uniform ferritic steel (test-blanket modules)
- Mirror: symmetric, except for field errors
  - Symmetry breaking added (min B) for interchange stability




ELM/RWM coils



ITER: test blanket modules



tandem mirror

# The progression of 3D/symmetry-breaking in fusion: stellarators, reversed field pinch

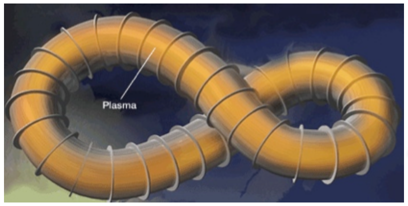
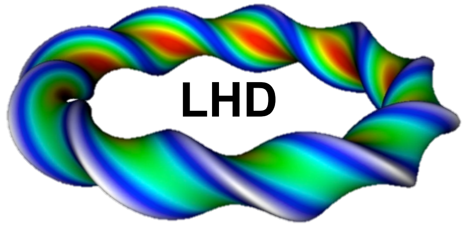


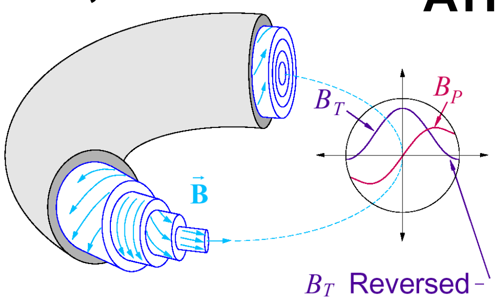

figure-8 stellarator

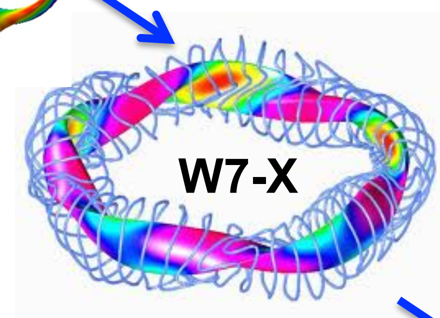


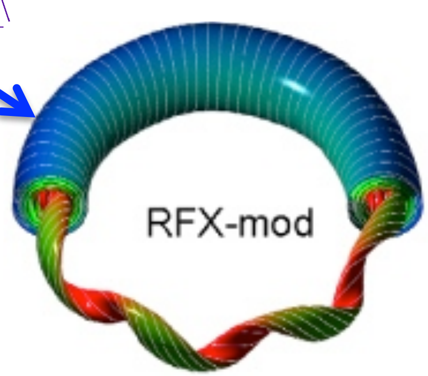

- Stellarators strongly 3D by design
- Areas of past/ongoing improvement
  - Avoidance of resonant field errors
  - Island suppression: shear, $\bar{\iota} \neq$ low order n/m
  - Confinement optimization
- Reversed field pinches – spontaneous helical states

ATF

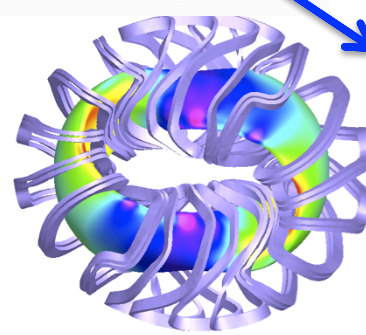

MST, RFX



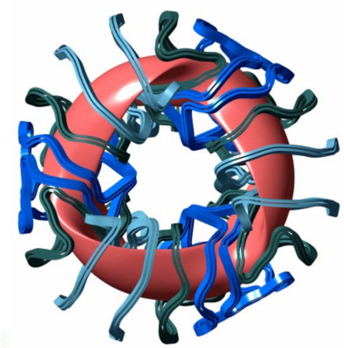




Self-organized helical state



QPS



NCSX

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# Symmetry-breaking effects likely to remain and lead to new physics issues

- **Reactors**
  - Coils further from plasma, but ports/non-uniformity likely in surrounding ferritic steel structures
  - 3D coils for control, rotational transform, optimization ➡ low re-circulating power
- **How much symmetry-breaking can be tolerated?**
  - Up to some level masked by turbulence, collisions, ambipolar $E_r$ field, island suppression by flows, etc.
  - Lower collisionality regimes of reactors => effects in current experiments may not be reactor compatible
    - Sufficient confinement to maintain H-mode pedestal profiles
    - Neoclassical toroidal viscosity – rotation effects
  - Energetic particle confinement, localized wall heat fluxes, lowered ignition margin
- **Advances in 3D simulation tools and diagnostics essential**

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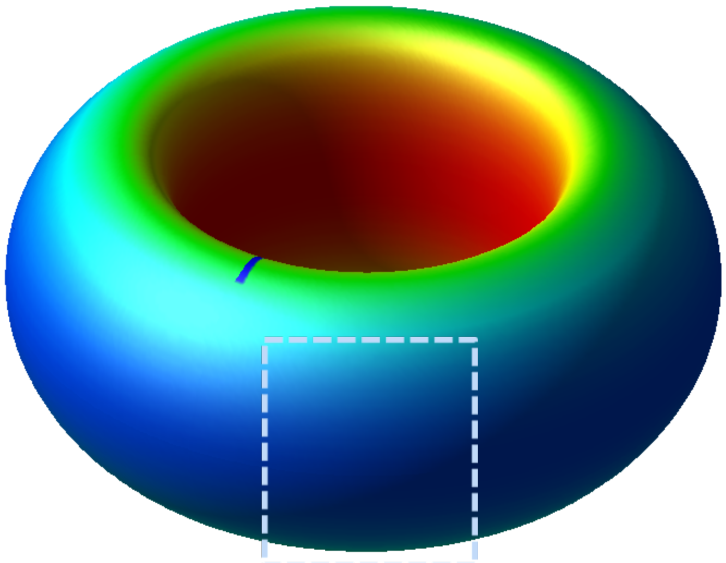
# Outline

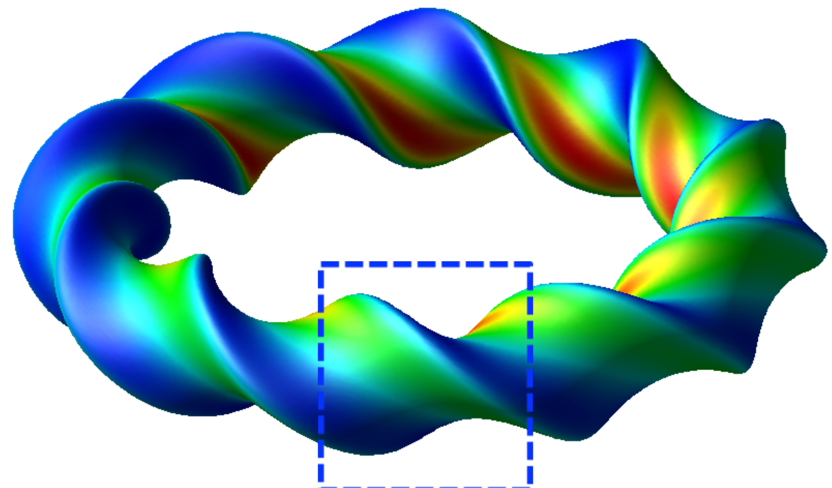
- **Characteristic forms of symmetry for 3D systems**
  - Field period symmetry, stellarator symmetry
  - Quasi-symmetry in magnetic coordinates (Hamada, Boozer)
  - Approximate degrees of deviation from axisymmetry
    - **3D tokamak** ($\delta B_{n\neq0}/\delta B_{n=0}$ = $10^{-3}$ to $10^{-2}$)
    - **Helical reversed field pinch state** ($\delta B_{n\neq0}/\delta B_{n=0}$ = 0.03 to 0.05)
    - **Stellarator** ($\delta B_{n\neq0}/\delta B_{n=0}$ = 0.1 to 0.3)
  - Resonant ($\delta B_{\perp} \neq 0$, $m = nq$) vs. non-resonant ($\delta B_{\perp} \sim 0$, $m \neq nq$)
- **3D design**
- **Equilibrium**
- **Confinement, transport**
- **Stability**
- Many new 3D theory/modeling approaches under development, cannot cover all in this talk

# Basic symmetries for 3D systems:

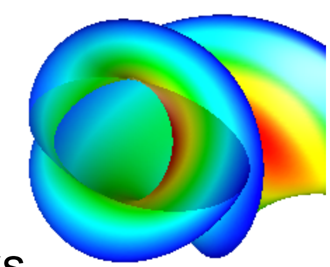
**Continuous symmetry:**
same view after arbitrary rotation ($N_{fp} \rightarrow \infty$)
magnetic field is integrable – Hamiltonian
with an ignorable coordinate



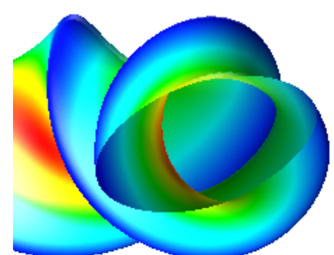
**Field period symmetry:**
equivalent view after a
discrete $2\pi/N_{fp}$ rotation
e.g. = 36° for LHD ($N_{fp}$ = 10)



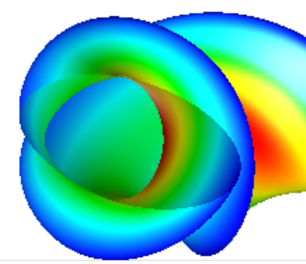
**Stellarator symmetry:**
Turn around and stand on your
head for an equivalent view
Simplifies analysis:
$R, B \sim \cos(m\theta - n\zeta)$ [$\cancel{\sin(m\theta - n\zeta)}$ ]
maintained in stellarators, but
broken in up-down asymmetric tokamaks



Turn around



Turn around
+vertical inversion

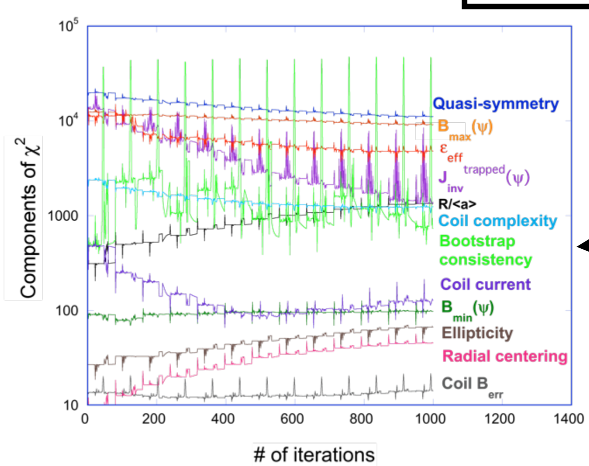


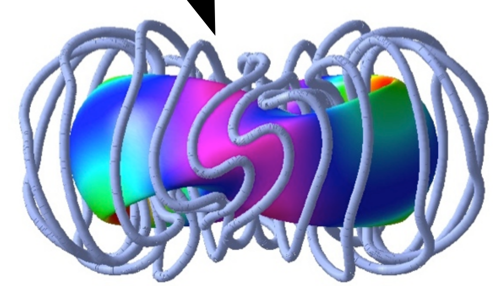
# Synthesis of 3D configurations

- Coils ➡ outer magnetic surface shape ➡ physics properties
- 3D shapes open up very large design space: ~ 40 independent parameters (A. Boozer, L. P Ku, 2010) based on SVD analysis
- Axisymmetric tokamak shape parameters: $\varepsilon, \kappa, \delta$
- Thought experiment: quantize shape parameters into 10 levels
  - $10^3$ 2D configurations vs. $10^{40}$ 3D configurations => "combinatorial explosion"
  - Other large numbers: $7 \times 10^{22}$ visible stars, $6 \times 10^{30}$ prokaryotes (bacteria) on earth's surface
- STELLOPT:

Adjust plasma boundary or coil geometry shape → Solve 3D equilibrium → Calculate $\chi^2$ (physics + engr. targets)

Levenberg-Marquardt or other method used to minimize $\chi^2$



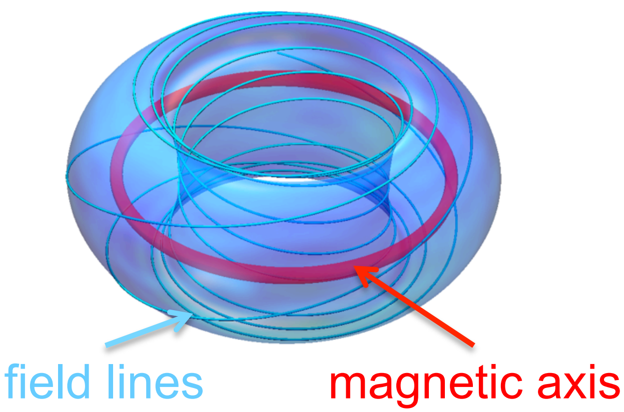



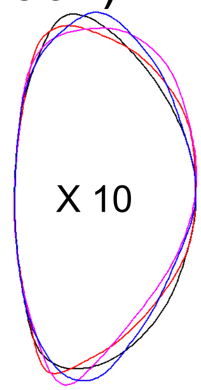
# 3 methods for creating rotational transform

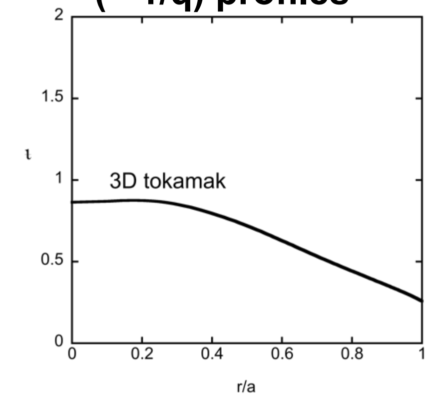
(L. Spitzer, 1951; C. Mercier, 1964)

**Rotational transform (= 1/q) profiles**

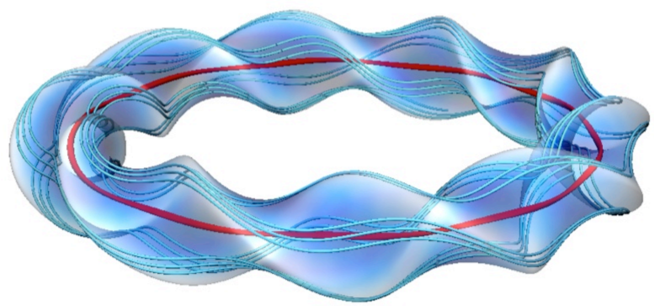
- Plasma current (3D tokamak)

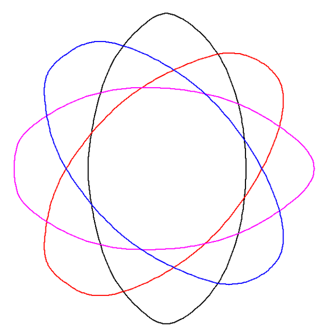


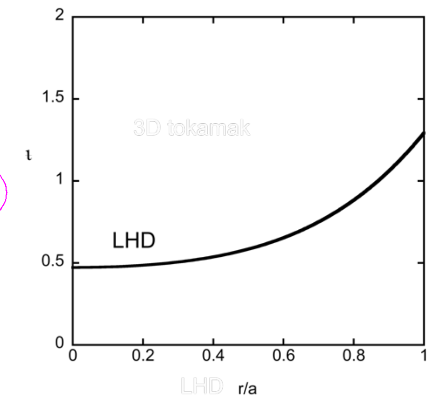



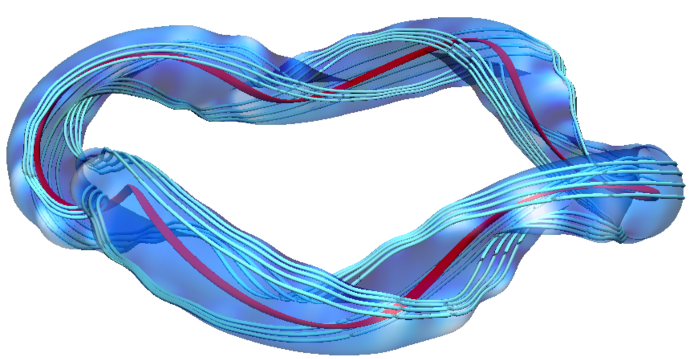

- Planar axis, rotating cross-section (LHD, ATF)

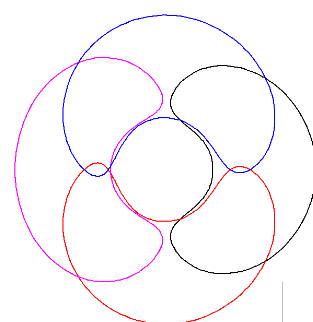


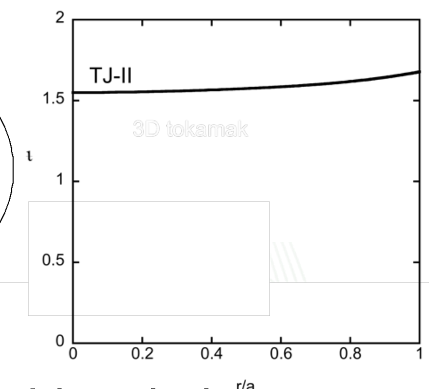





- Helical magnetic axis (TJ-II, HSX, W7-X, H1-NF)

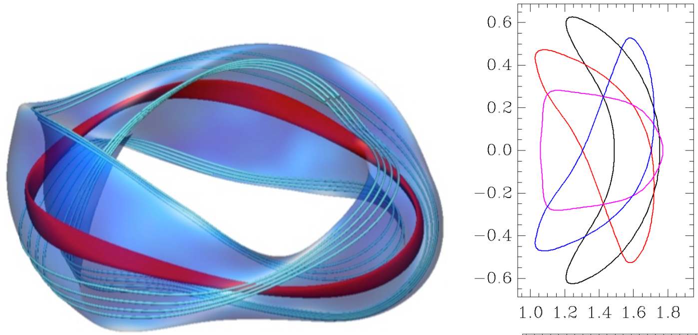


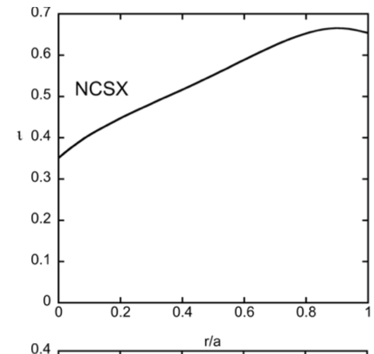




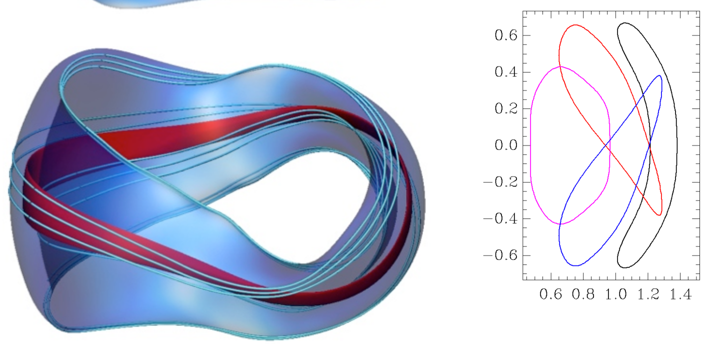

Configuration scaling: $\iota\!\!\!-/ N_{fp}$ = const., $\langle R\rangle/\langle a\rangle/ N_{fp}$ = const., $N_{fp}$ = field periods

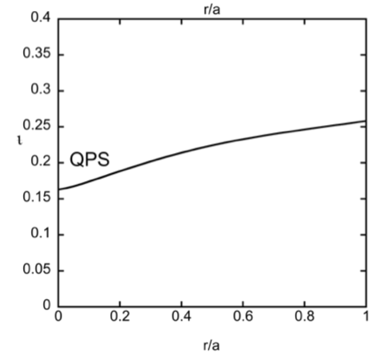
- **Stellarator/tokamak hybrids**
  - NCSX





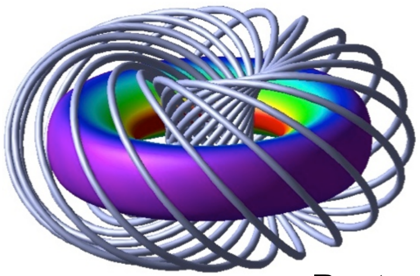

  - QPS



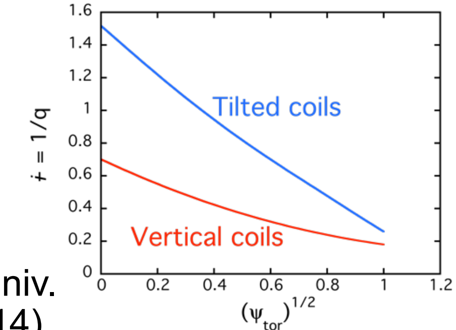



  - Tilted coils

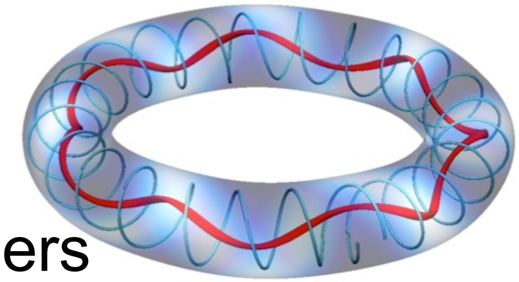
  transform amplifier

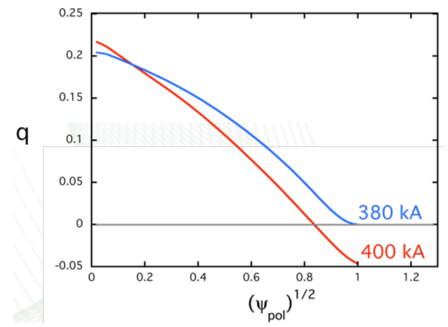


**Proto-CIRCUS** - Columbia Univ.
A. Clark, F. Volpe, et al. (2014)



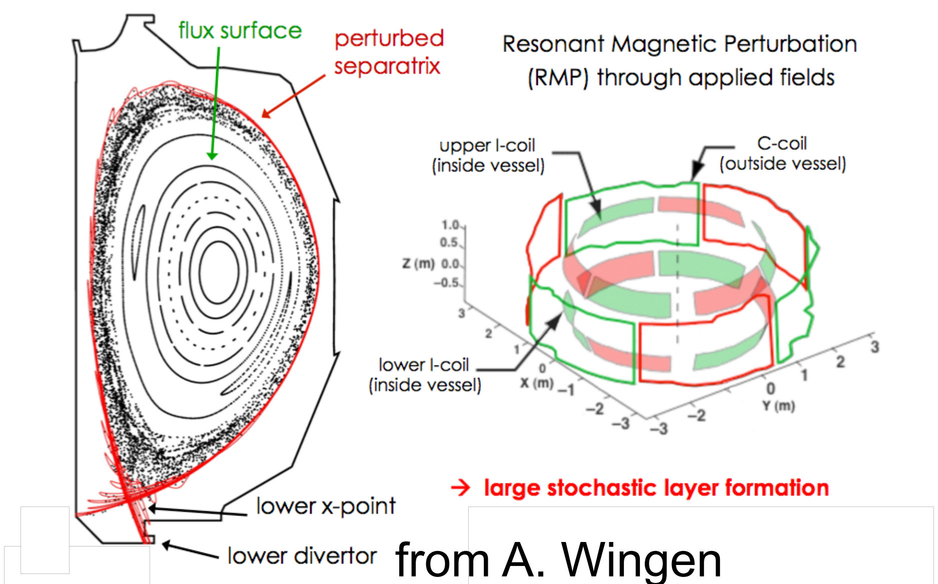

- **Reverse field pinches**
  - single helicity states
  - sustainment, transport barriers





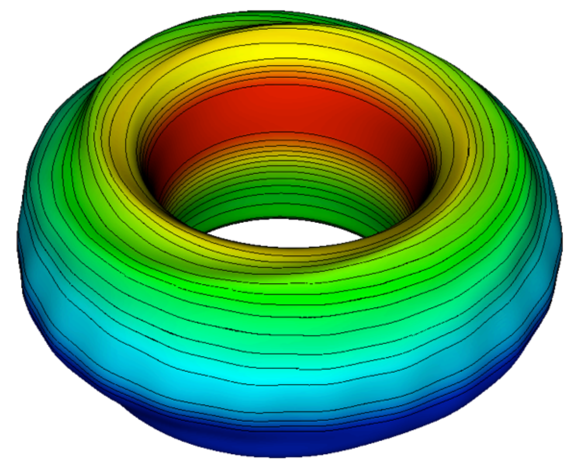
# 3D tokamaks: ELM/RWM controls

- Window-pane coils ➡ produce magnetic field $\perp$ to outer flux surfaces (~$10^{-3}$ of axisymmetric field)
  - Resonant ($m = nq$) and non-resonant ($m \neq nq$) fields present
  - Goal is to locally break up outer flux surfaces
    - Islands to limit pedestal region
    - suppress edge-localized mode (ELM) instabilities
- TF ripple, test blanket modules, non-symmetric ferritic steel

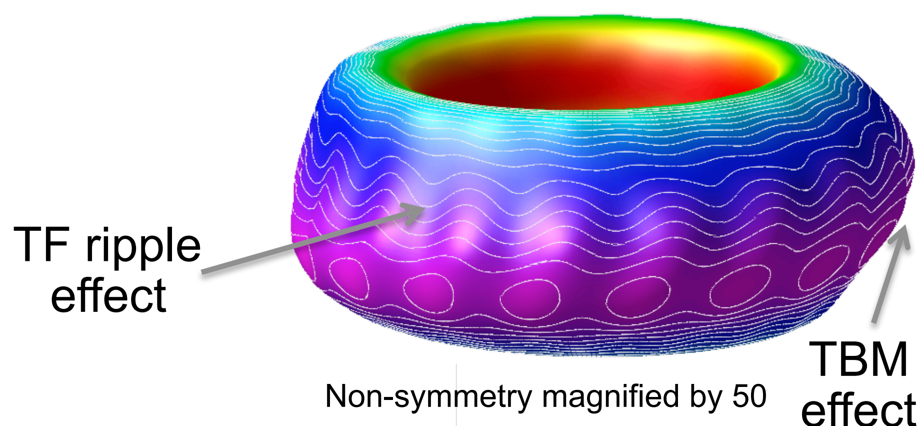



from A. Wingen

Resonant fields - ELM coil



Non-resonant fields – TF ripple/TBM



# The 3D toroidal equilibrium challenge

- Basic equations
  - force balance $\vec{F} = \vec{\nabla} p - \vec{J} \times \vec{B} = 0$
  - Ampere's law $\mu_0 \vec{J} = \vec{\nabla} \times \vec{B} \Rightarrow \vec{\nabla} \bullet \vec{J} = 0$ ➡ $W = \int \left( \frac{|B|^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d^3x$
  - absence of monopoles $\vec{\nabla} \bullet \vec{B} = 0$

Variational principle
(Kruskal, Kulsrud, 1958)

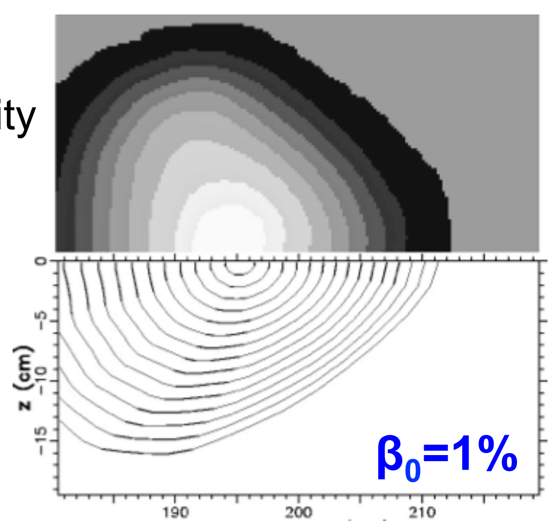
- Fundamental issues
  - Non-existence of 3D nested surface equilibria (Grad, 1967; Lortz, 1971) except for closed field line systems
  - Axisymmetry ➡ Grad-Shafranov equation (nonlinear elliptic PDE)
  - 3D ➡ mixed nonlinear hyperbolic/elliptic system (also occurs in transonic flow, lower hybrid wave propagation, Weitzner, 2014)
  - Singular current sheets at rational surfaces, islands, chaotic field lines

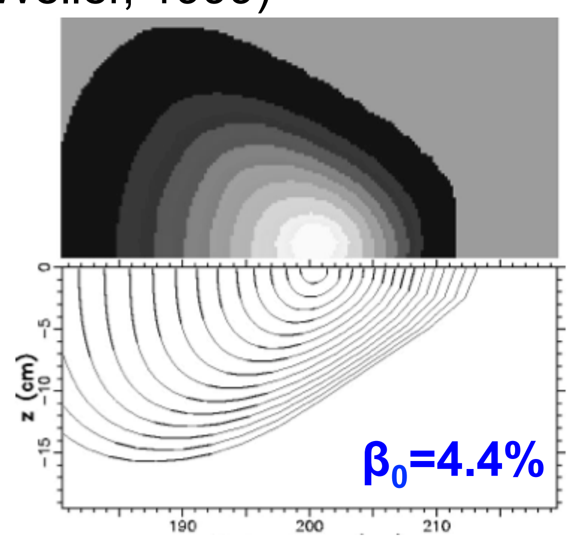
# Nested flux surface approach: VMEC (Variational Moments Equilibrium Code)

- **Constrains magnetic field lines to lie on nested flux surfaces** [Hirshman, Whitson (1983)] $\vec{B} = \vec{\nabla}\zeta \times \vec{\nabla}\Phi_{pol} + \vec{\nabla}\theta \times \vec{\nabla}\Phi_{tor}$ $\zeta, \theta = \text{toroidal/poloidal angles}, 2\pi\Phi_{pol,tor} = \text{mag. fluxes}$
- **Steepest-descent minimization of variational form and force balance**
  – Finds equilibria without fully resolving singular currents
  – generally good approximation to the more exact case with islands
  – Stellarators avoid large islands by magnetic shear or avoiding low order rational $\iota\!\!-$
- **Inverse solution, solves for:** $\mathrm{R}=\sum R_{mn}(\Phi_{tor})\cos(m\theta - n\zeta);\ Z=\sum Z_{mn}(\Phi_{tor})\sin(m\theta - n\zeta)$
- **Computationally efficient: used for many 3D physics calculations, stellarator optimization (STELLOPT), 3D reconstruction (V3FIT)**
- **Validation on W7-AS stellarator** (A. Weller, 1999)

X-ray emissivity contours

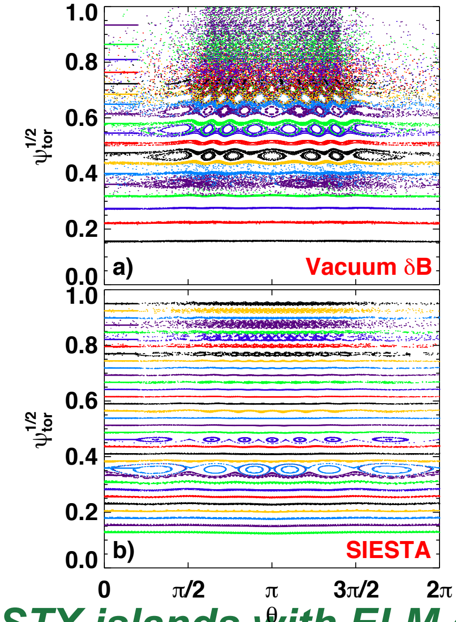
Calculated flux surfaces



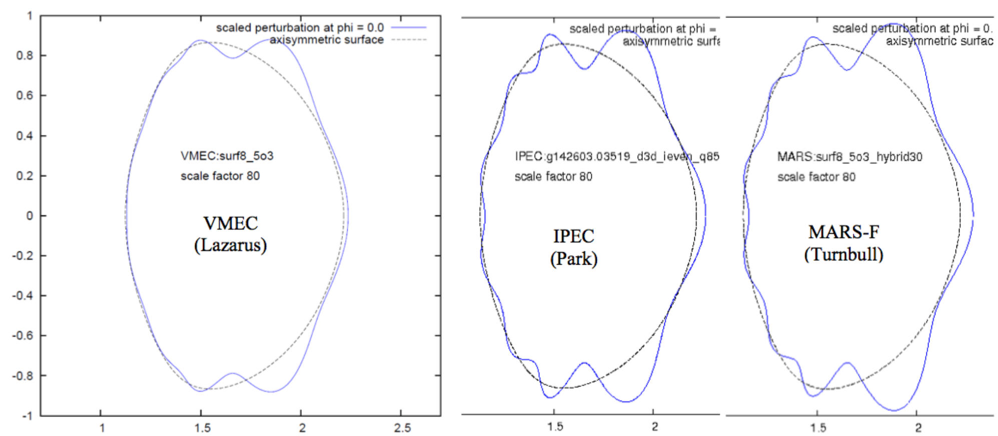
# 3D equilibria with broken flux surfaces

- **Nonlinear equilibrium solvers**
  - VMEC ➡ SIESTA (direct solver)
    - allows for component of B normal to initial surfaces, also, pressure evolves
  - PIES (A. Reiman, 1986), HINT-2 (Y. Suzuki, 2006)
    - direct iterative solvers
- **Linearized MHD (takes into account rotation, 2-fluid effects, dissipation)**
  - Magnetic islands stagnate flows, requires transfer of torque
  - M3D-C$^1$ (N. Ferraro)
  - MARS-F (A. Turnbull)
  - IPEC (J.-K. Park)
- **Other approaches**
  - Discontinuous pressure
    - SPEC (S. Hudson)
    - dp/dr = 0 on rational surfaces
  - Superposition
    - Vacuum 3D field on 2D equilibrium



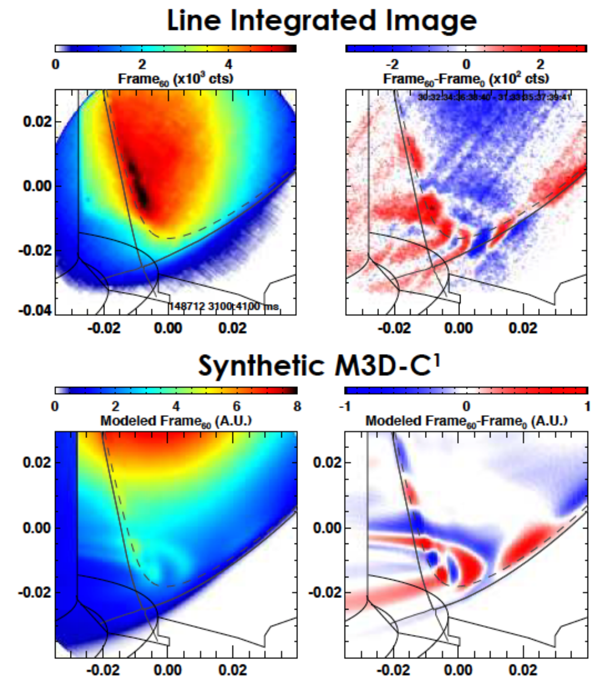

***NSTX islands with ELM coils***
*J. Canik (2013)*

***q = 8.5/3 flux surface comparison***
***perturbation scaled by factor of 80***
*(E. Lazarus, J.-K.Park., A. Turnbull, A. Reiman, 2014)*
***J. D. King, APS/DPP invited talk TI1.04***



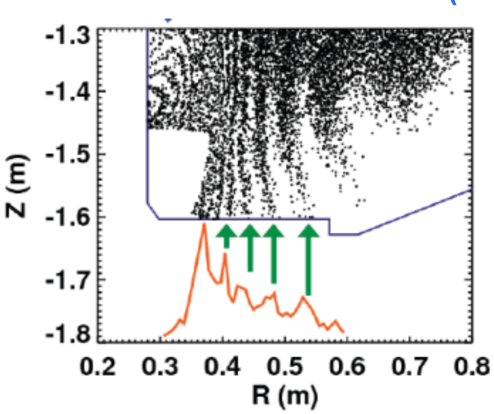
# 3D tokamak edge: corrugation, kinks, or peeling/ballooning?

- **New diagnostics**
  - Soft X-ray emissivity measurement
  - Energy filtering➡image analysis➡reconstruction
- **Transport, stability effects**
  - Rotation braking, density pump-out
  - Mode-locking thresholds
  - Island screening vs. amplification
  - Maintain wall/scrape-off-layer separation
- **Divertor effects**
  - Strike point splitting, homoclinic tangles
  - Loss of detachment with 3D field application



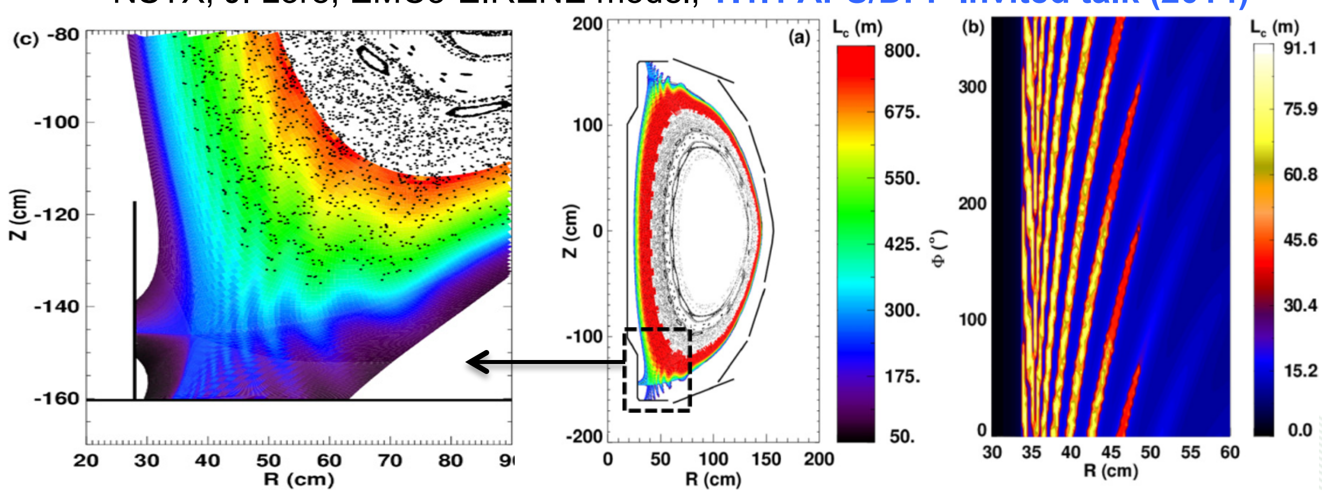

DIII-D, N. Ferraro, Z. Unterberg, A. Wingen, M. Shafer, et al. (2013)

GI1.3 APS/DPP Invited talk (2014)



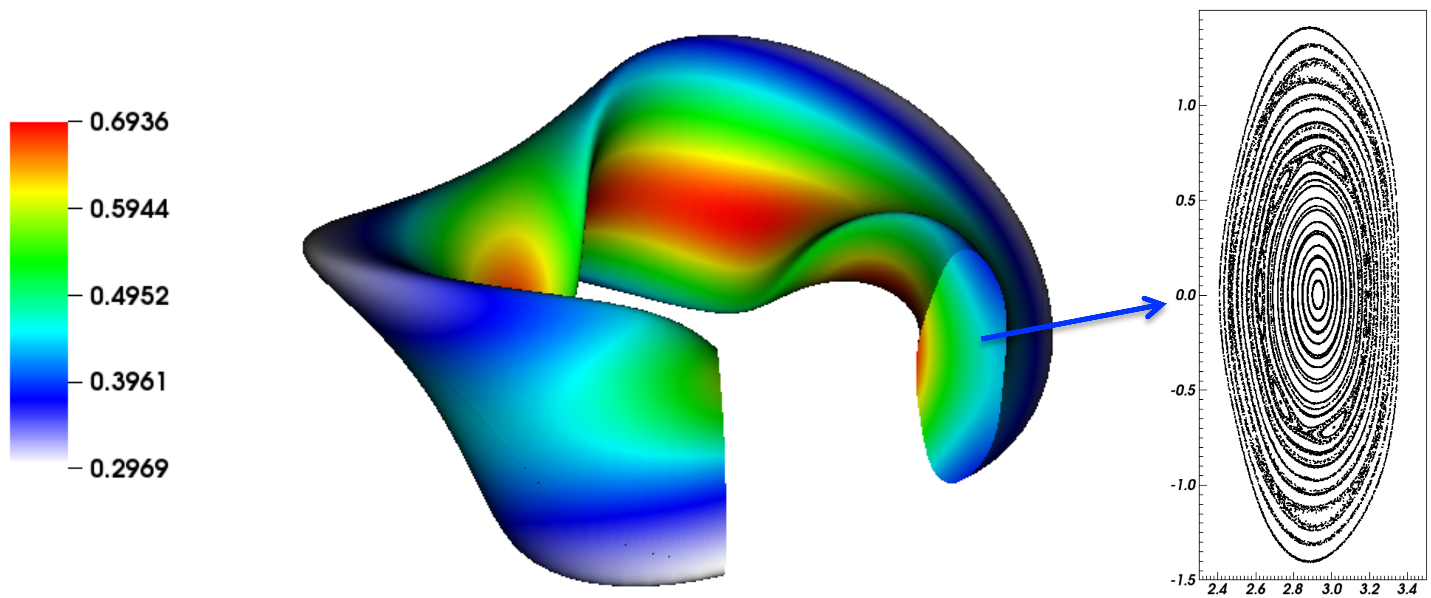

*J-W. Ahn, Nucl. Fusion 50, 045010 (2010).*

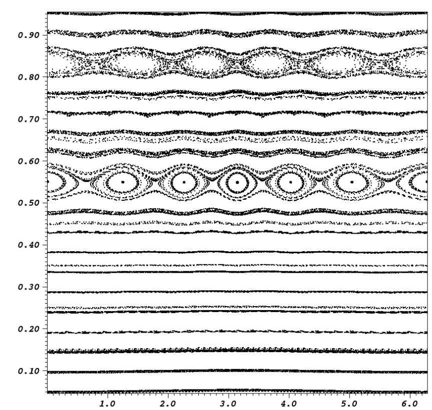
NSTX, J. Lore, EMC3-EIRENE model, TI1.1 APS/DPP Invited talk (2014)



# Application of SIESTA to stellarators

- Due to the lack of a continuous symmetry, stellarators have regions with island structures
- NCSX/QPS optimization experience showed these islands can be minimized by coil design
- Original motivation for the development of SIESTA: provide a rapidly evaluated target function for island minimization
- Also important for physics/transport modeling

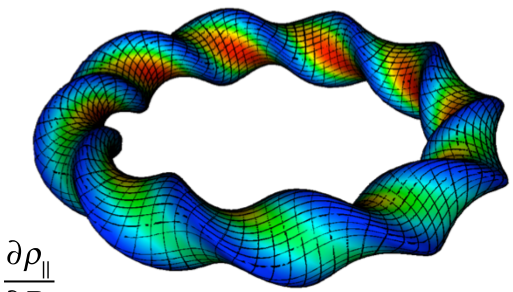


Island chain in compact stellarator

# Particle Orbits in 3D fields

- Guiding-center
  - Canonical coordinates (A. Boozer, R. White, 1981)
    - Only involves |B| and currents in straight field line coordinates



$$\frac{d\psi}{dt} = \frac{1}{D}\left(I\frac{\partial B}{\partial \zeta} - g\frac{\partial B}{\partial \theta}\right)\left(\mu + \frac{mv_{\parallel}^2}{B}\right); \qquad \frac{d\rho_{\parallel}}{dt} = \frac{\bar{\iota} - \rho_{\parallel} g'}{D}\dot{P}_{\theta} + \frac{\rho_{\parallel} I'}{D}\dot{P}_{\zeta}$$

$$\frac{d\theta}{dt} = \left[\left(\mu + \frac{mv_{\parallel}^2}{B}\right)\frac{\partial B}{\partial \psi} + e\frac{\partial \Phi}{\partial \psi}\right]\frac{\partial \psi}{\partial P_{\theta}} + eBv_{\parallel}\frac{\partial \rho_{\parallel}}{\partial P_{\theta}}; \qquad \frac{d\zeta}{dt} = \left[\left(\mu + \frac{mv_{\parallel}^2}{B}\right)\frac{\partial B}{\partial \psi} + e\frac{\partial \Phi}{\partial \psi}\right]\frac{\partial \psi}{\partial P_{\zeta}} + eBv_{\parallel}\frac{\partial \rho_{\parallel}}{\partial P_{\zeta}}$$

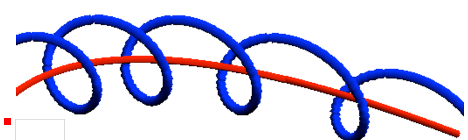
  - Non-canonical coordinates (Littlejohn, Cary 1979-83)
    - Lie-transform perturbation methods, variational action integral
    - Coordinate-free

$$\frac{d\vec{R}}{dt} = \frac{1}{B_{\parallel}^*}\left(\mathrm{v}_{\parallel}\vec{B}^* + \vec{E}^* \times \hat{b} + \frac{\mu\hat{b}\times\vec{\nabla}B}{Ze}\right); \quad \frac{d\mathrm{v}_{\parallel}}{dt} = Ze\left(\vec{E}^* + \vec{R}\times\vec{B}^*\right)\mu\vec{\nabla}B$$

$$\vec{B}^* = \vec{B} + \frac{m\mathrm{v}_{\parallel}}{Ze}\vec{\nabla}\times\hat{b}; \quad \vec{E}^* = \vec{E} - \frac{m\mathrm{v}_{\parallel}}{Ze}\frac{\partial\hat{b}}{\partial t}$$

- Lorentz equation: $\dfrac{d\vec{\mathrm{v}}}{dt} = \dfrac{Ze}{m}\left(\vec{E} + \vec{\mathrm{v}}\times\vec{\mathrm{B}}\right)$

G.C.

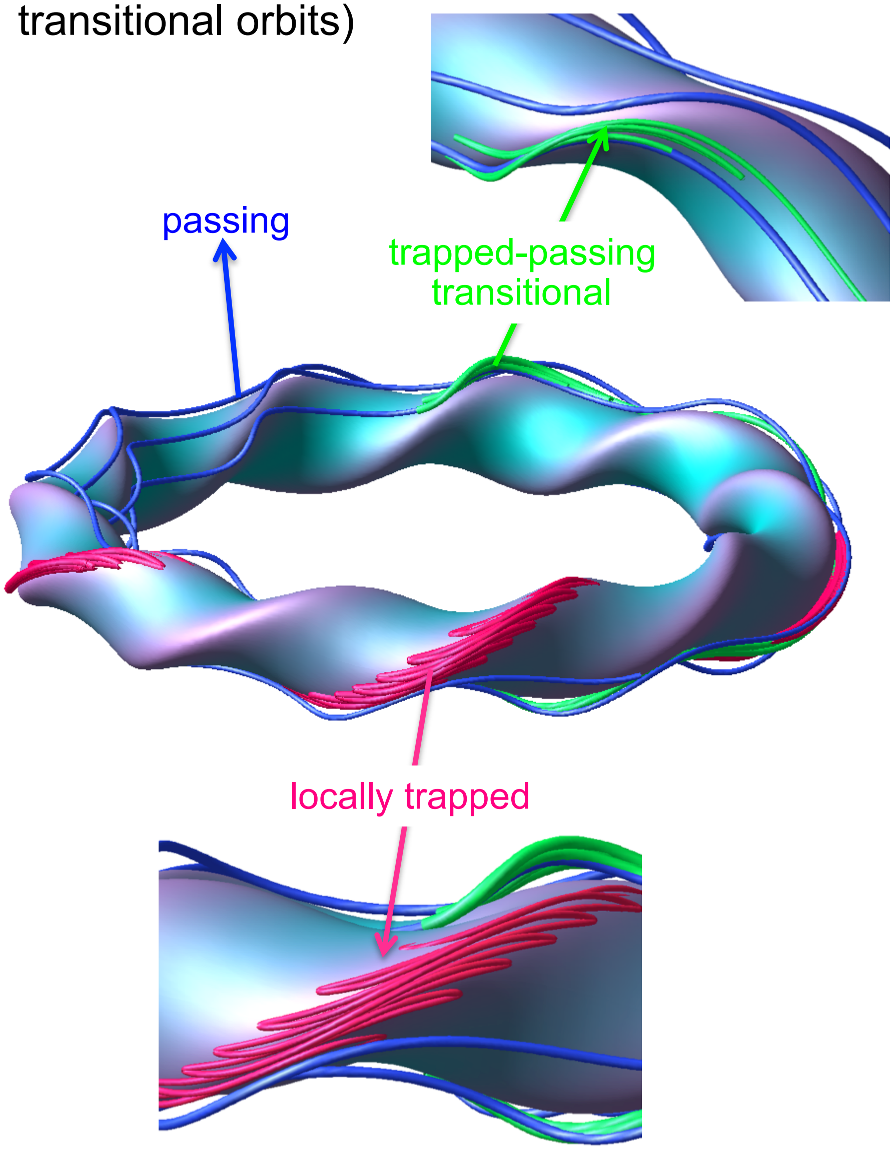


**Issues for simulations:** energy conservation, Liouville's theorem (conservation of phase space volume carried with particle), intersections of fast ion with walls, PFCs

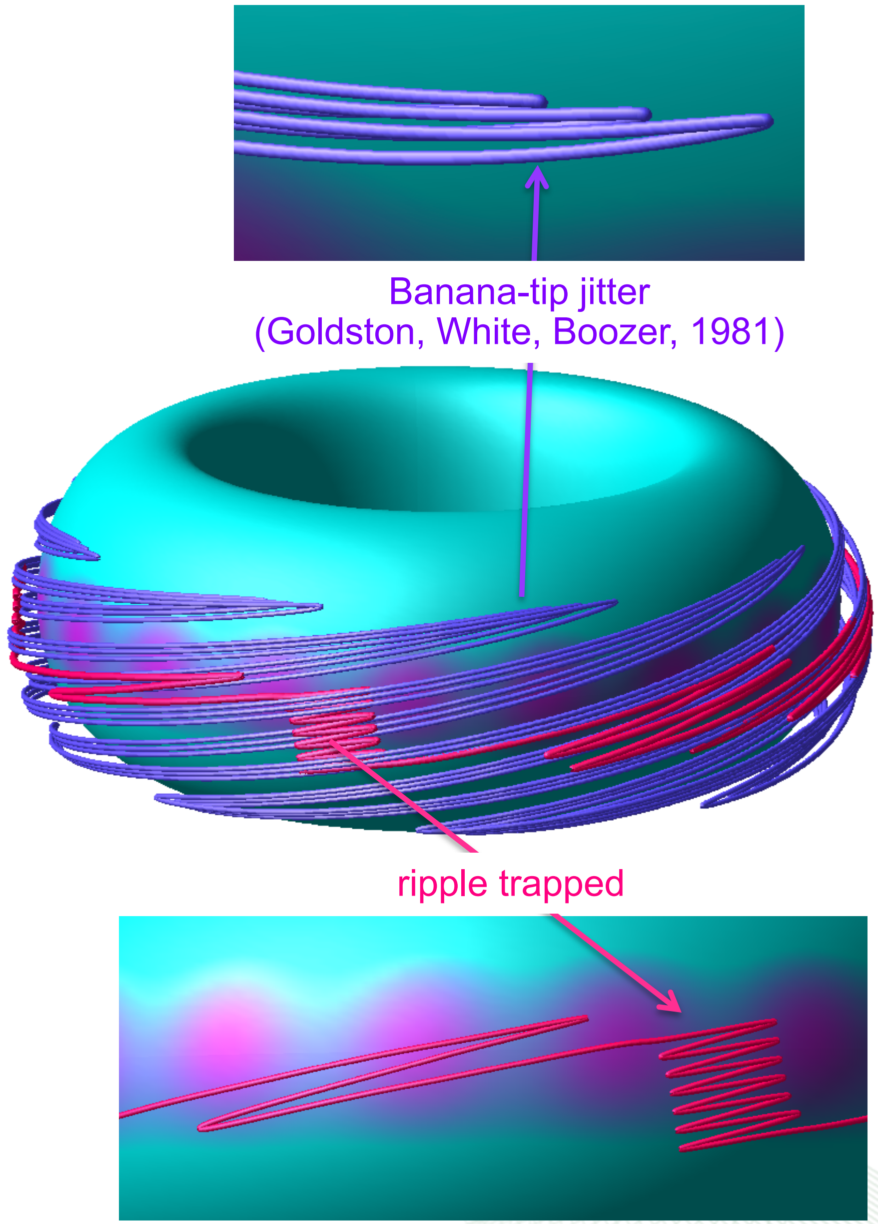
$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial z^i}\left(\sqrt{g}\frac{\partial z^i}{\partial \tau}\right) = 0$$

# Particle trajectories in 3D configurations: many new classes of orbits

Stellarator ($E_r$ added for confinement of trapped and transitional orbits)

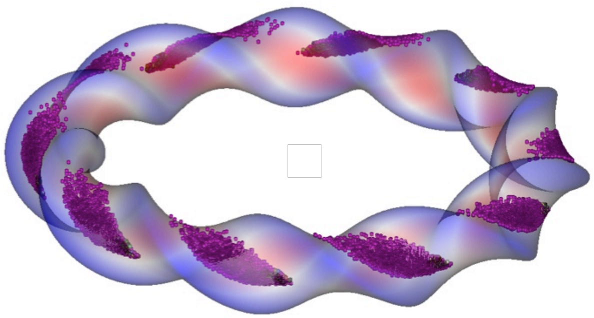


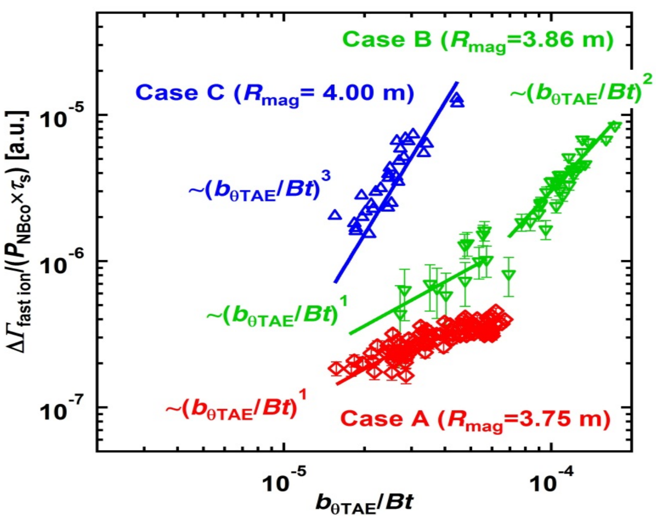

Tokamak with ripple and TBM



# Particle Monte Carlo simulations used extensively for energetic particle confinement studies in 3D systems

## Stellarator neutral beam transport

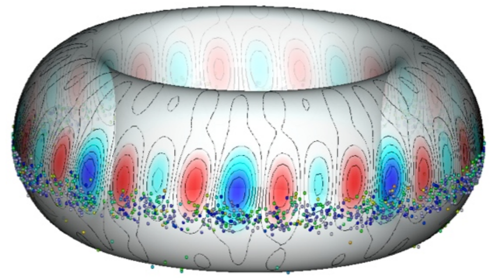




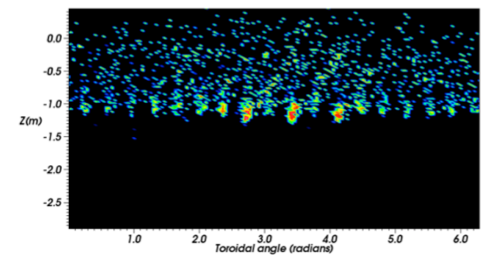

Loss scaling in LHD – K. Ogawa

## Tokamaks with 3D perturbations

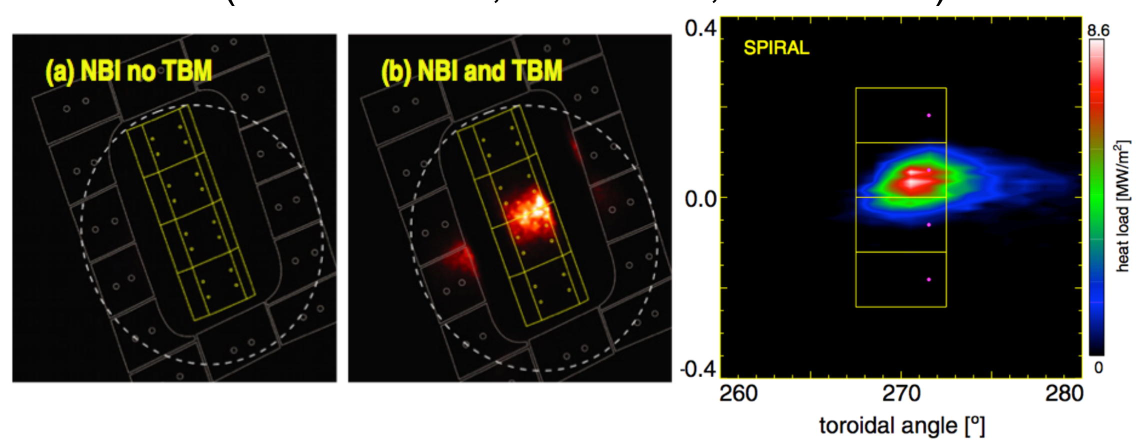
Fast ion exit locations ITER (TF+TBM)



Wall heat load localization (D. Spong 2011)




Fast ion losses in DIII-D with TBM coils (SPIRAL code, G. Kramer, 2011-2013)




**RWM coils:** ASCOT and SPIRAL show EP losses go to divertor plates

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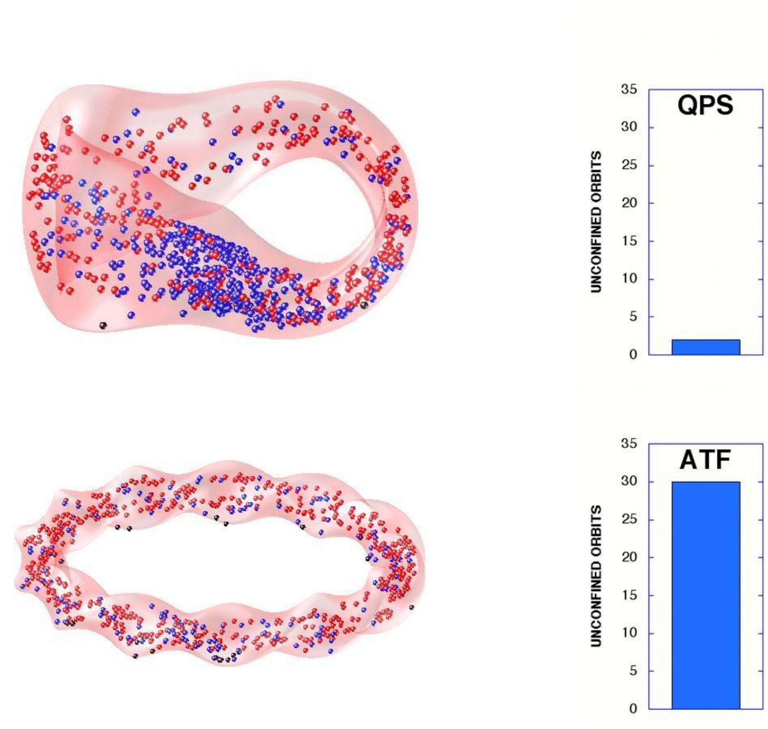
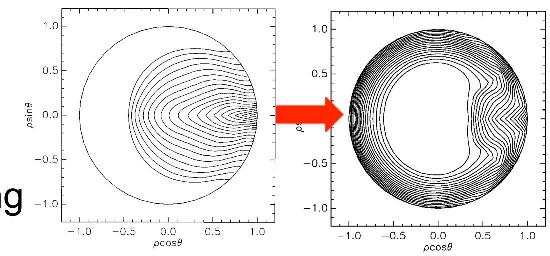
# Orbit characteristics have also been a dominant factor in stellarator optimization




- Quasi-symmetry B = B(ψ,MΘ-Nζ)
  Nührenberg, Zille (1988)
  - Dual meaning: (1) hidden, (2) approximate
  - Quasi-helical (M,N integers)/toroidal (N=0)/poloidal (M=0)
  - Transport isomorphic to an tokamak
- Quasi-omnigeneity
  - $J = \oint v_{\|}\, dl = J(\psi)$

  Constant |B| contour spacing

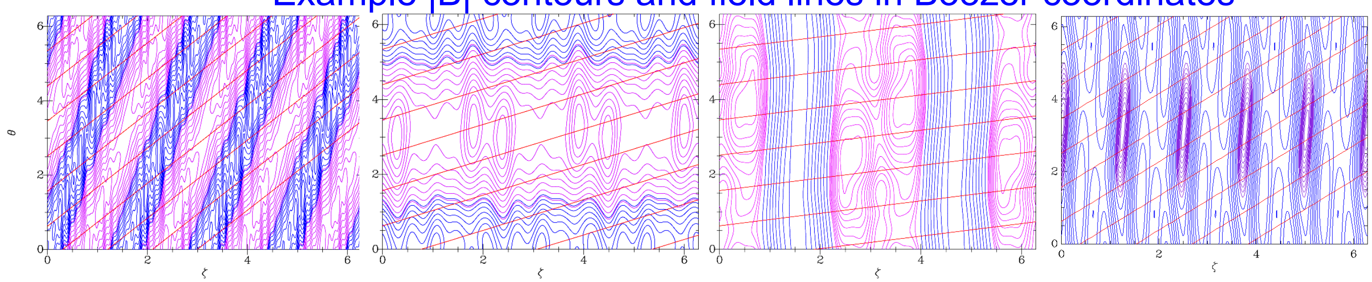
  Cary, Shasharina (1997)

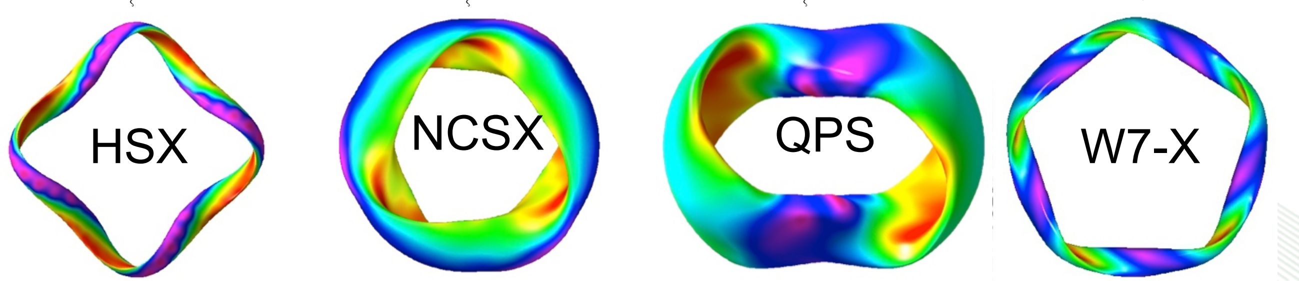



*D. Spong, S. Hirshman, et al. (STELLOPT, 1998)*

- Quasi-isodynamic
  - Poloidally closed |B| contours
- $B_{min}$ and $B_{max}$
  - Min/max along field line
  - Const. on flux surface
  - Deeply trapped: $B_{min}(\psi)$
  - Transitional: $B_{max}(\psi)$

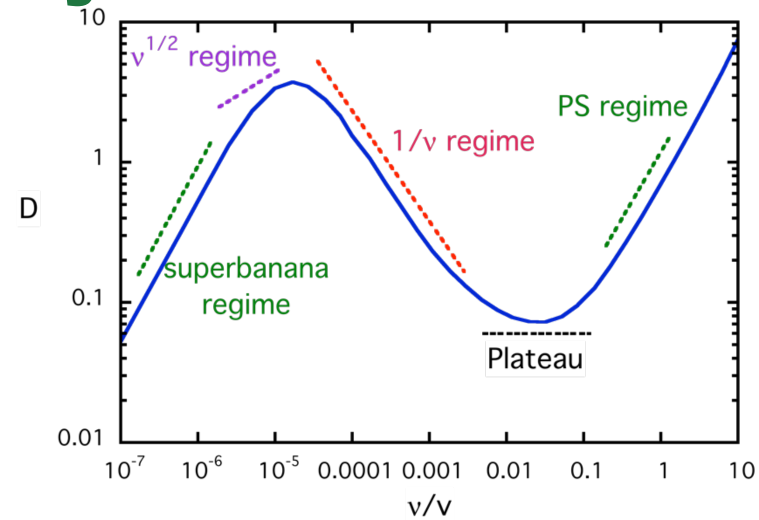
Example |B| contours and field lines in Boozer coordinates



# Collisional transport in 3D systems

- Characteristics unique to 3D
  - ripple transport regime ($1/\nu$) - trapped particle uncompensated radial drifts
  - ambipolarity condition: $\Gamma_{\text{ion}} = \Gamma_{\text{elelctron}}$ only for specific $E_r$
  - Stronger dependence on $E_r$ than tokamak
  - Bootstrap current can be suppressed or reversed
  - Flows in direction of highest symmetry
  - Collisionality and $E_r$ variation – low ripple moves $1/\nu$ to lower collisionality

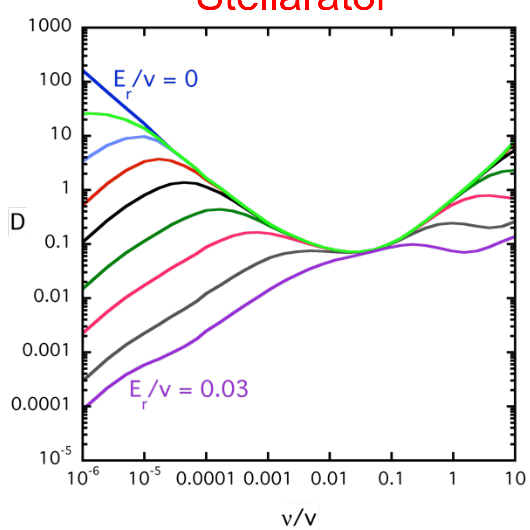



$D \propto \mathrm{v}_d^2 / \nu$ Uncompensated radial trapped drifts

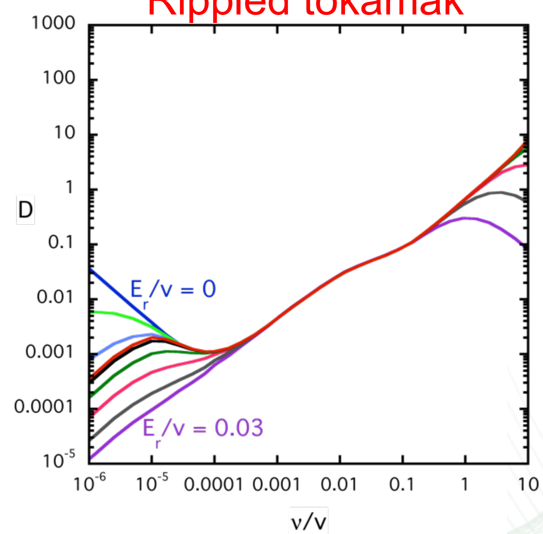
$D \propto \mathrm{v}_d^2 \left( RB / E_r \right)^{3/2} \nu^{1/2}$ Trapping/detrapping from collisions

$D \propto \mathrm{v}_d \left( \mathrm{v}_d RB / E_r \right)^2 \nu$ Trapping/detrapping from drifts

Stellarator



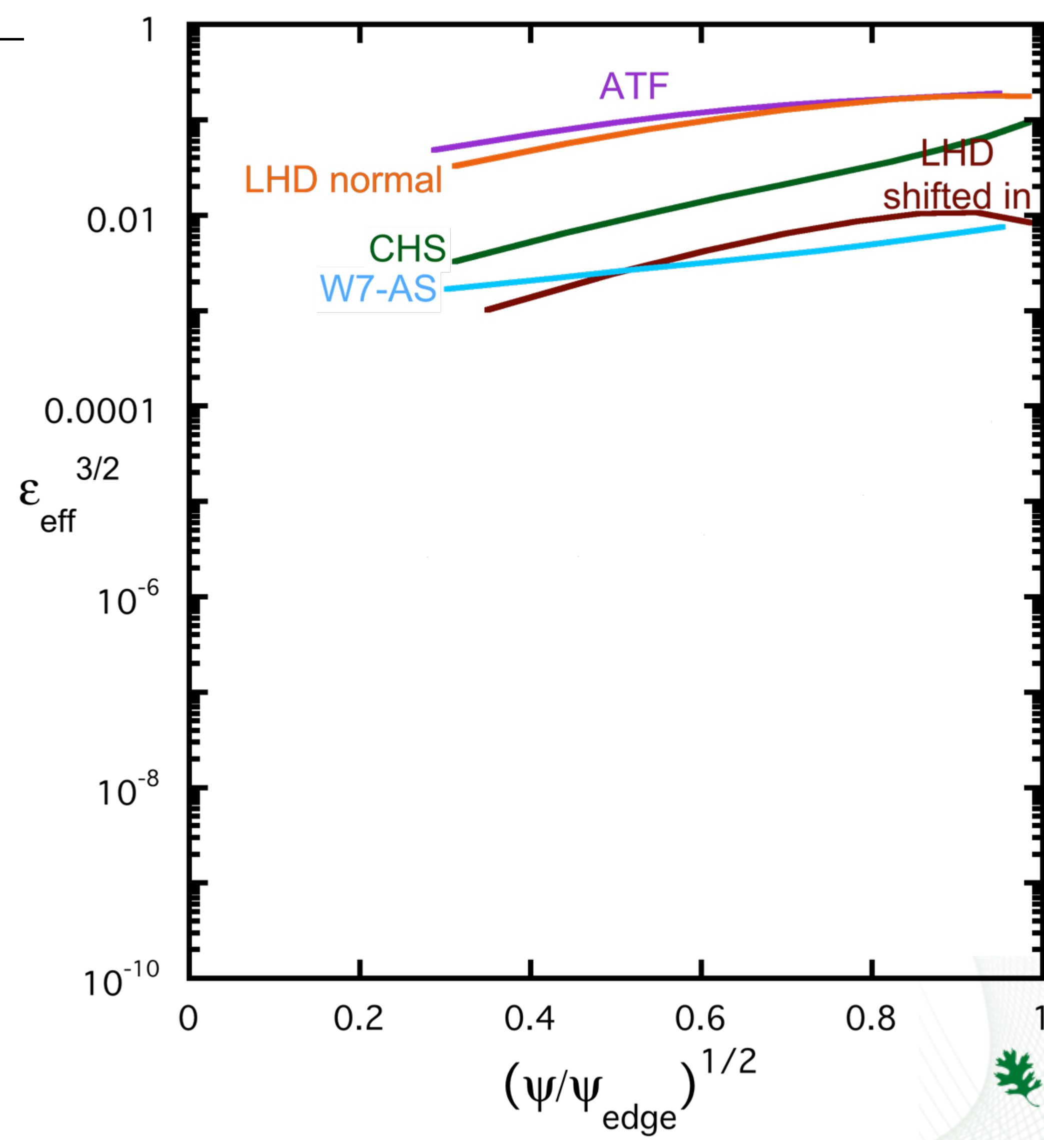

Rippled tokamak



## Useful characterization of ripple transport levels: effective ripple parameter

- $D_{1/\nu} / D_{plateau} = \left(\frac{4}{3\pi}\right)^2 \frac{(2\varepsilon_{eff})^{3/2}}{\nu^*}$
- Nemov, Kasilov,Kernbichler (1999)
- $\varepsilon_{eff}$ = 0 for ideal tokamak, quasi-symmetry, or quasi-omnigeneity
- Simple measure of orbit deviations from ideal

conventional stellarators

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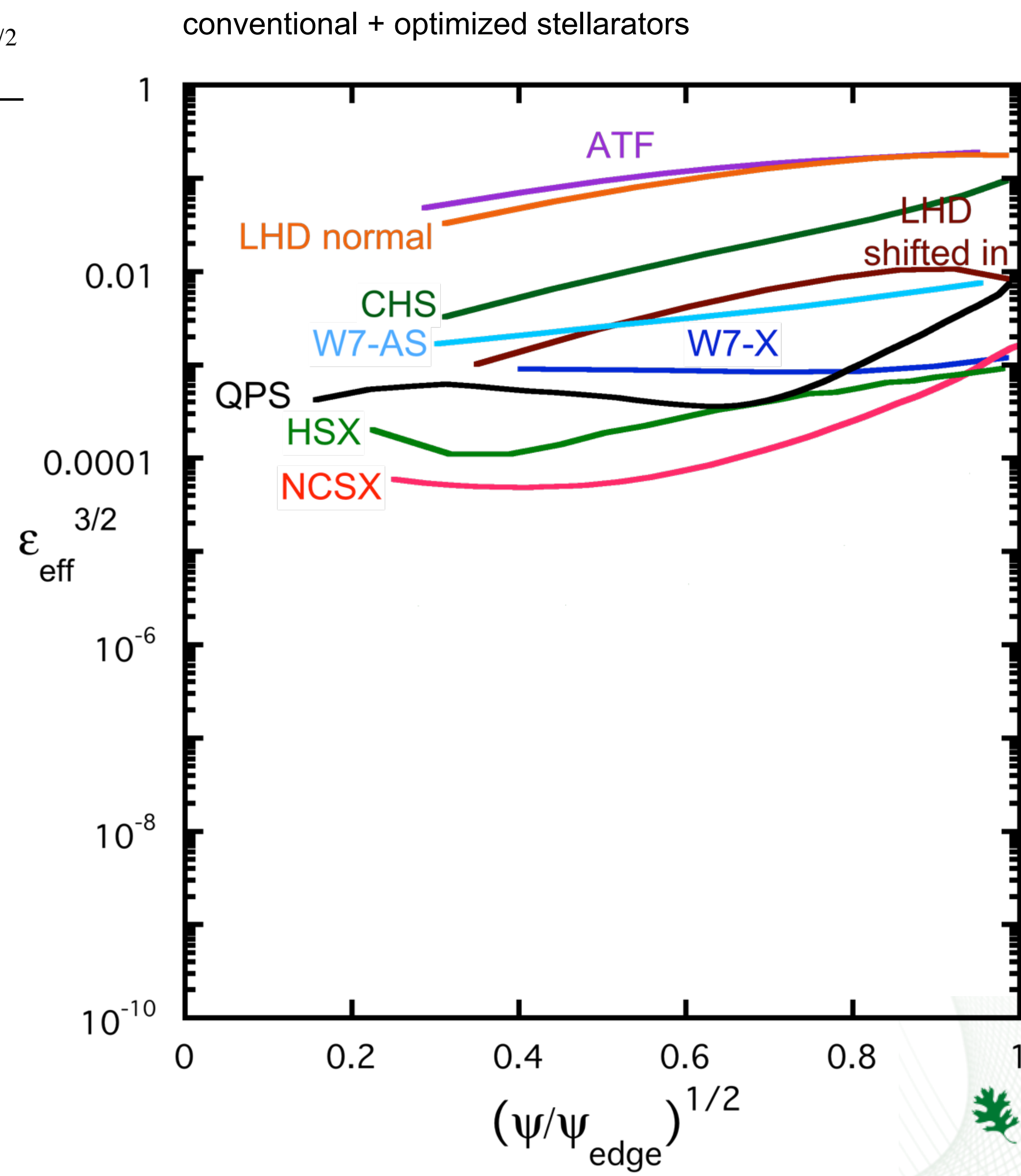

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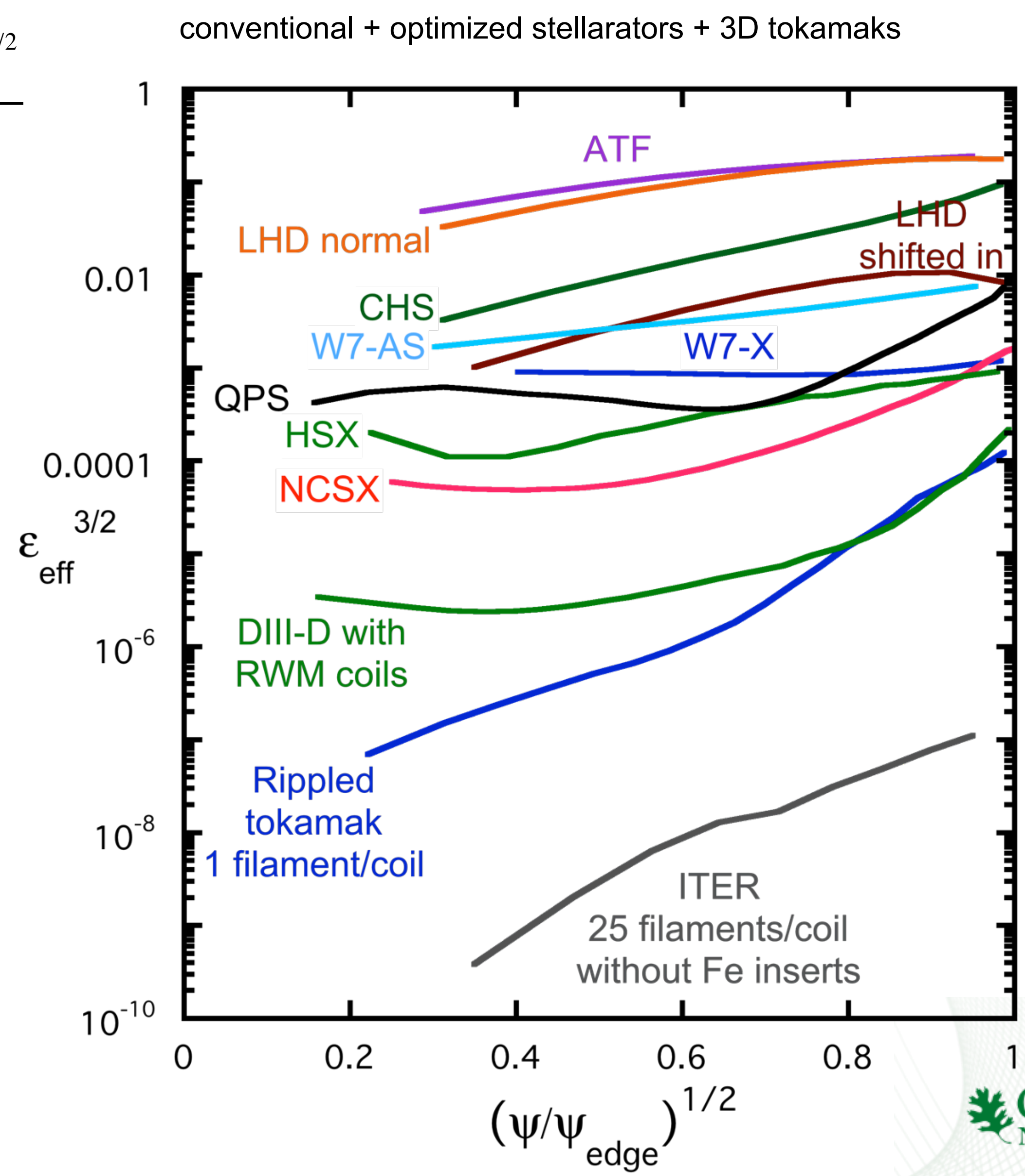


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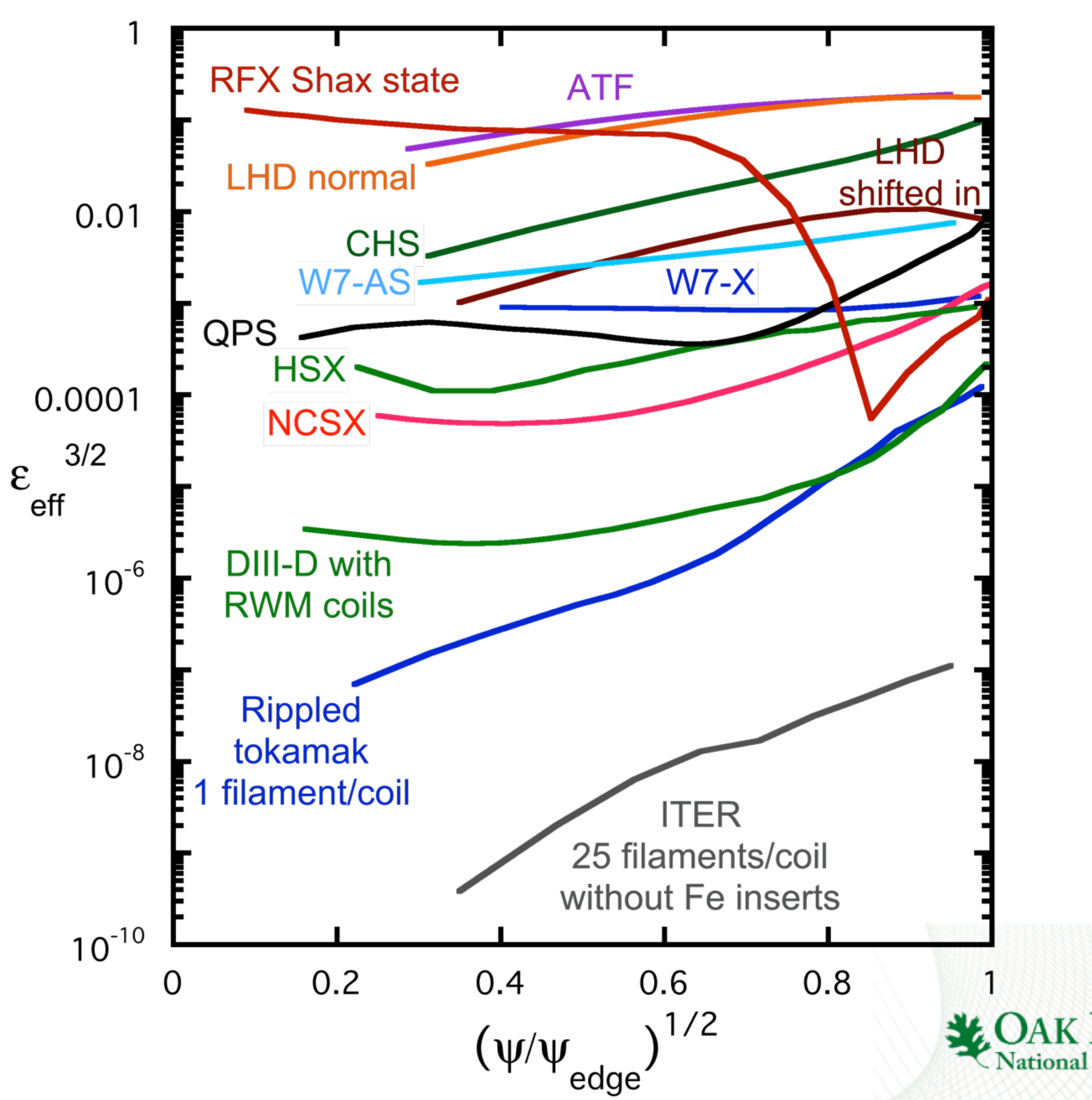
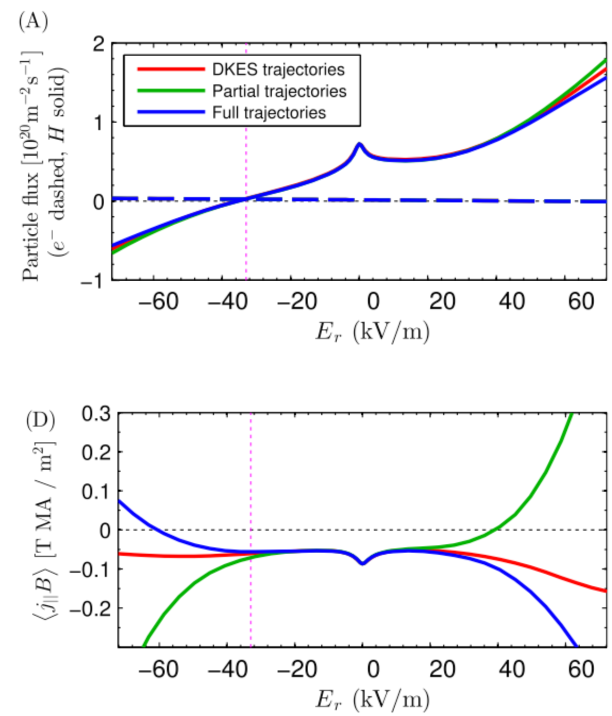
conventional + optimized stellarators + 3D tokamaks + helical RFPs



# Transport coefficient calculations in 3D systems

- Drift Kinetic Equation Solver (DKES) W. van Rij, S. Hirshman (1989)
  - 3D: toroidal/poloidal angle, $v_{||}/v$ – energy and radius, parameters, pitch angle scattering only, incompressible E x B approximation: $\vec{E}\times\vec{B}/B^2 \approx \vec{E}\times\vec{B}/\langle B^2\rangle$
- Ripple-averaged GSRAKE (C. Beidler, et al., 1995)
- δf Monte Carlo (MOCA, VENUS, FORTEC-3D) – verification study (C. Beidler, et al., NF, 2011)
- Moments method: correction for momentum non-conservation in DKES coefficients (H. Sugama, S. Nishimura, 2002; M. Taguchi, 1992)
- 4D (toroidal/poloidal angle, $v_{||}/v$, energy SFINCS) M. Landremann (2014)
  - Momentum conserving collisions, full drift trajectories, local diffusive in radius
- Impurity transport with $\phi = \phi(\rho,\theta,\zeta)$ (J.M. García-Regaña, 2013; C. Beidler, 1995)
- 3D tokamak NTV (neoclassical toroidal viscosity) J. D. Callen, IAEA-2010, K. Shaing, 2003-10 $\delta B/B < \rho_{ion}/a$ regime

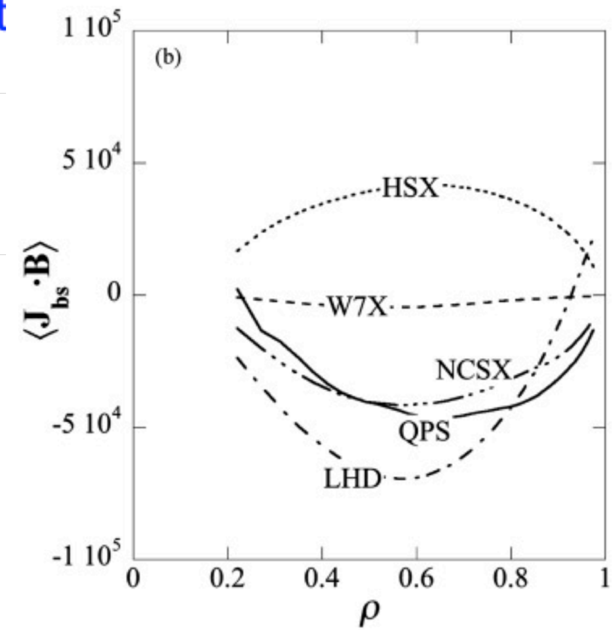



**DKES/SFINCS** comparisons
for W7-X, M. Landremann (2014)
**APS/DPP 2014 Invited talk BI1.5**

# PENTA model – self-consistent (ambipolar) flows, currents with momentum corrections from Sugama, Nishimura method

Radial particle flows required for ambipolar condition ⇨ self-consistent energy fluxes and bootstrap current

$$\begin{bmatrix} \Gamma_e \\ q_e/T_e \\ \Gamma_i \\ q_i/T_i \\ J_{BS}^E \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{11}^{ei} & L_{12}^{ei} & L_{1E}^{e} \\ L_{21}^{ee} & L_{22}^{ee} & L_{21}^{ei} & L_{22}^{ei} & L_{2E}^{e} \\ L_{11}^{ie} & L_{12}^{ie} & L_{11}^{ii} & L_{12}^{ii} & L_{1E}^{i} \\ L_{21}^{ie} & L_{22}^{ie} & L_{21}^{ii} & L_{22}^{ii} & L_{2E}^{i} \\ L_{E1}^{e} & L_{E2}^{e} & L_{E1}^{i} & L_{E2}^{i} & L_{EE} \end{bmatrix} \begin{bmatrix} X_{e1} \\ X_{e2} \\ X_{i1} \\ X_{i2} \\ X_E \end{bmatrix}$$



A. Ware, FS&T (2006)

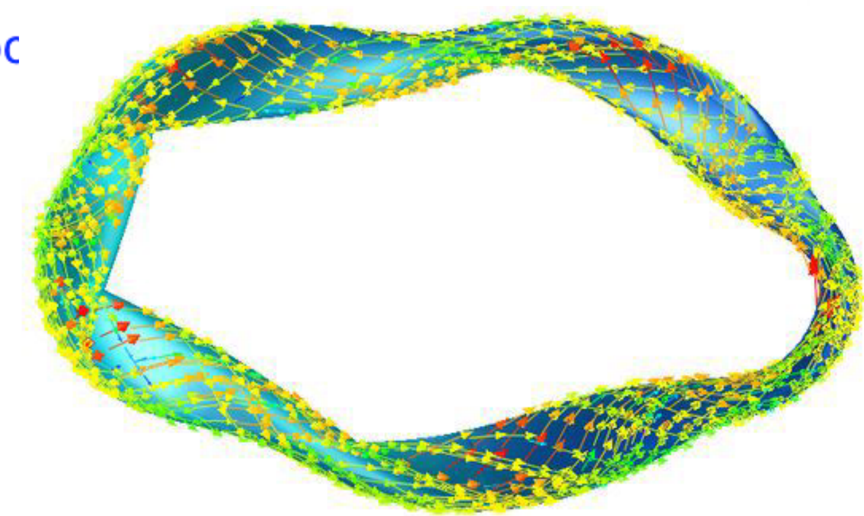
Parallel mass and energy flows:

$$\begin{bmatrix} \langle B u_{\parallel i} \rangle \\ \dfrac{2}{5 p_i} \langle B q_{\parallel i} \rangle \end{bmatrix} = \left[ \frac{1}{\langle B^2 \rangle} \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i2} & M_{i3} \end{bmatrix} - \begin{bmatrix} l_{11}^{ii} & -l_{12}^{ii} \\ -l_{12}^{ii} & l_{22}^{ii} \end{bmatrix} \right]^{-1} \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i2} & N_{i3} \end{bmatrix} \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}$$

Poloidal and toroidal (contra-variant) flow veloc

$$\begin{bmatrix} \langle u_i^\theta \rangle \\ \langle u_i^\zeta \rangle \end{bmatrix} = \frac{4\pi^2}{V'} \begin{bmatrix} \chi' & -B_\zeta / \langle B^2 \rangle \\ \psi' & B_\theta / \langle B^2 \rangle \end{bmatrix} \begin{bmatrix} \langle B u_{\parallel i} \rangle / \langle B^2 \rangle \\ X_{i1} \end{bmatrix}$$

$$\text{where } X_{a1} \equiv -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a \frac{\partial \Phi}{\partial s}, \quad X_{a2} \equiv -\frac{\partial T_a}{\partial s}, \quad X_E = \langle B E_\parallel \rangle / \langle B^2 \rangle^{1/2}$$



H. Sugama, S. Nishimura, 2002; M. Taguchi, 1992;
D. A. Spong, Phys. Plasmas, 2005

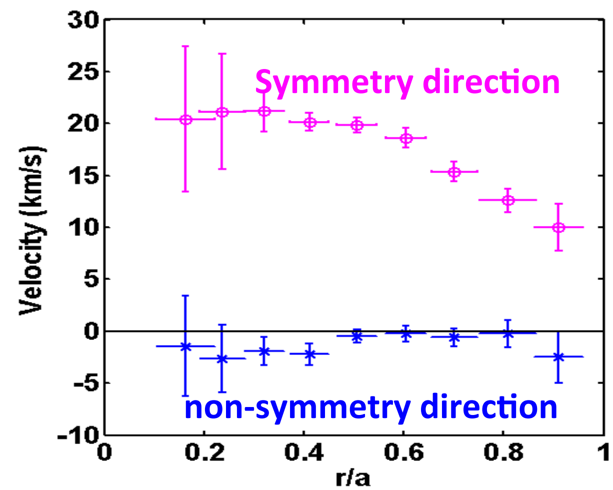
# HSX provides test of parallel neoclassical transport properties for quasi-helical symmetry

PENTA code calculates plasma flow over wide range of ripple $\varepsilon_{eff}$ :

tokamaks ⟶ rippled tokamaks ⟶ quasi-symmetric ⟶ conventional stellarators

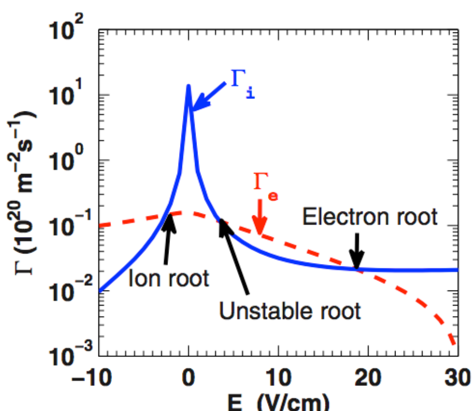
ITER ~$10^{-6}$ NSTX w/RMP ~$10^{-4}$ HSX ~3 x$10^{-3}$ LHD, TJ-II ~ 0.05 - 1

HSX: CXRS measure large flow in direction of quasi-symmetry

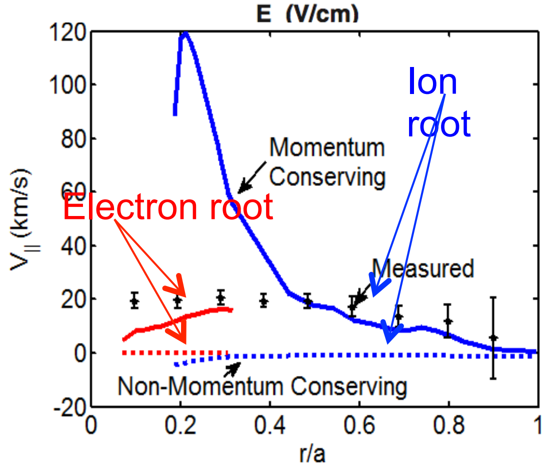


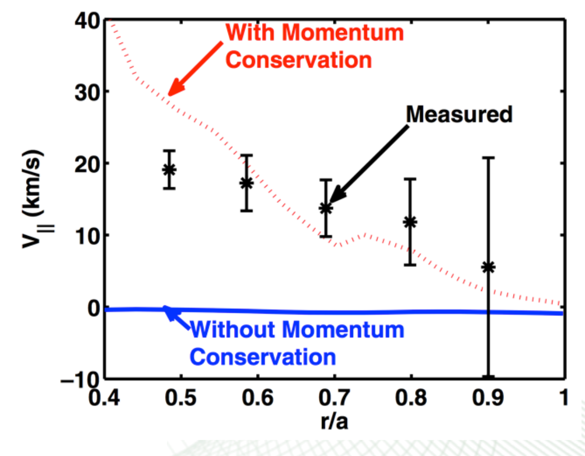

Parallel flows in HSX agree well with PENTA:

- Importance of momentum conservation
- Large sheared flows reduce turbulent transport



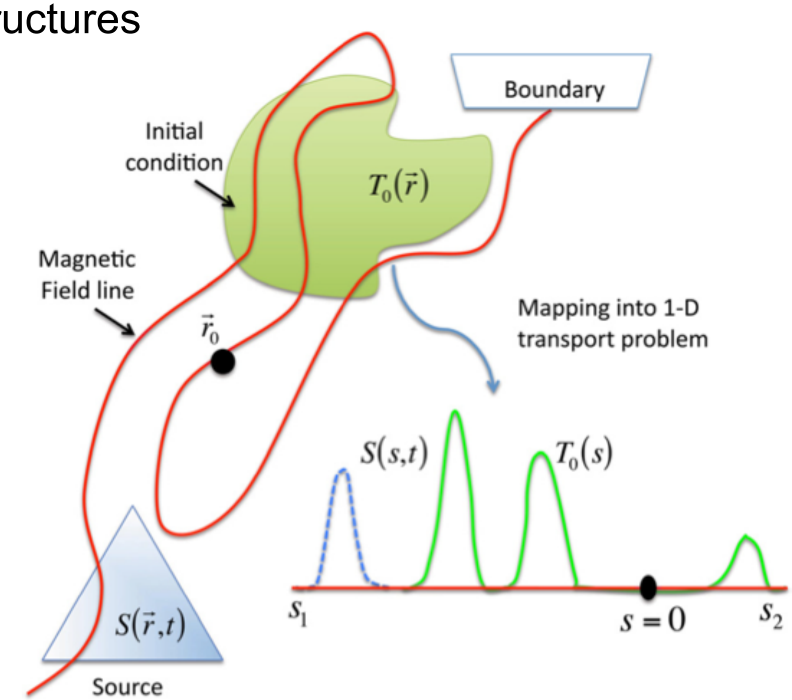

J. Lore, et al., Phys. Plasmas (2010)
A. Bresemeister, et al., PPCF (2013)



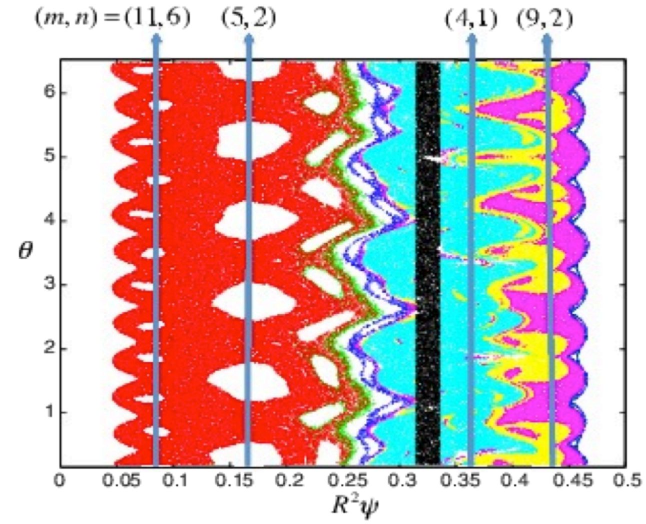



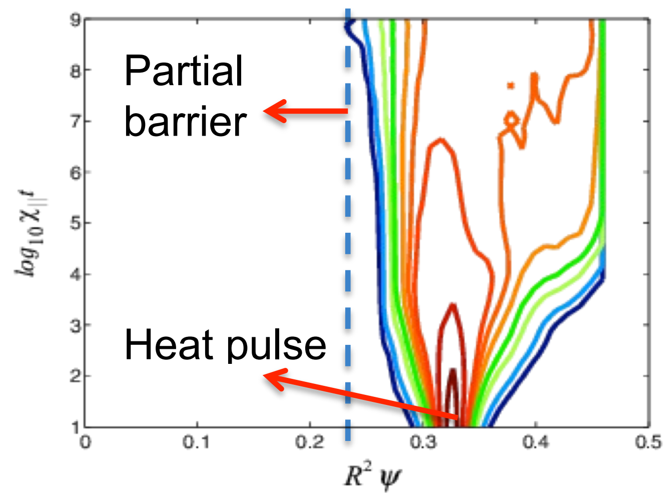
# Tokamak 3D edge transport with islands, chaotic regions

- Parallel transport dominates
  - $\chi_{||}/\chi_{\perp} \sim 10^9 - 10^{10}$
- LG (Lagrangian Green's function) method
  - D. del-Castillo-Negrete, L. Chacon (2012)
  - stable, high accuracy method for high $\chi_{||}/\chi_{\perp}$ regime
  - Can treat fields with complex filamentation/braiding
  - Reduced to solution of coupled 1D ODE's
  - Transport barriers possible even with chaotic structures




Partial heat transport barriers in the absence of magnetic flux surfaces




LG solution of heat transport equation in 3-D chaotic field

del-Castillo-Negrete, Blazevski Nucl. Fusion (2014)

# Stability issues for 3D configurations

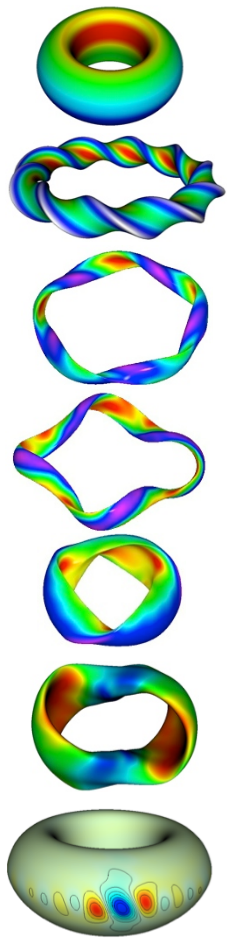
- **Energetic particle (EP) instabilities**
  - Development of EP global gyrokinetic models
  - Tokamak 3D edge effects in NSTX
- **Micro-turbulence**
  - Optimization of 3D systems for turbulent transport
- **3D tokamak edge**
  - Tearing/kink/ballooning/peeling
    - 3D perturbations can both suppress (DIII-D) and trigger (NSTX, ASDEX-upgrade) ELMs
  - Edge turbulence increase with 3D fields (G. McKee, et al., NF 2013)
  - Recent theory:
    - linear δW CAS3D stability (E. Strumberger, et al., NF, 2014)
    - 3D peeling-ballooning formulation (T. Weyens, R. Sanchez, et al., Phys. Plasmas, 2014)
- **High $\beta$ stellarator regimes**
  - Soft β limit, LHD $\beta_{peak}$ ~ 5%; W7-AS $\beta_{peak}$ ~ 7%
  - Second stable hybrid systems, β = 23%, $\beta_N$ = 19 (A. Ware, PRL, 2002)

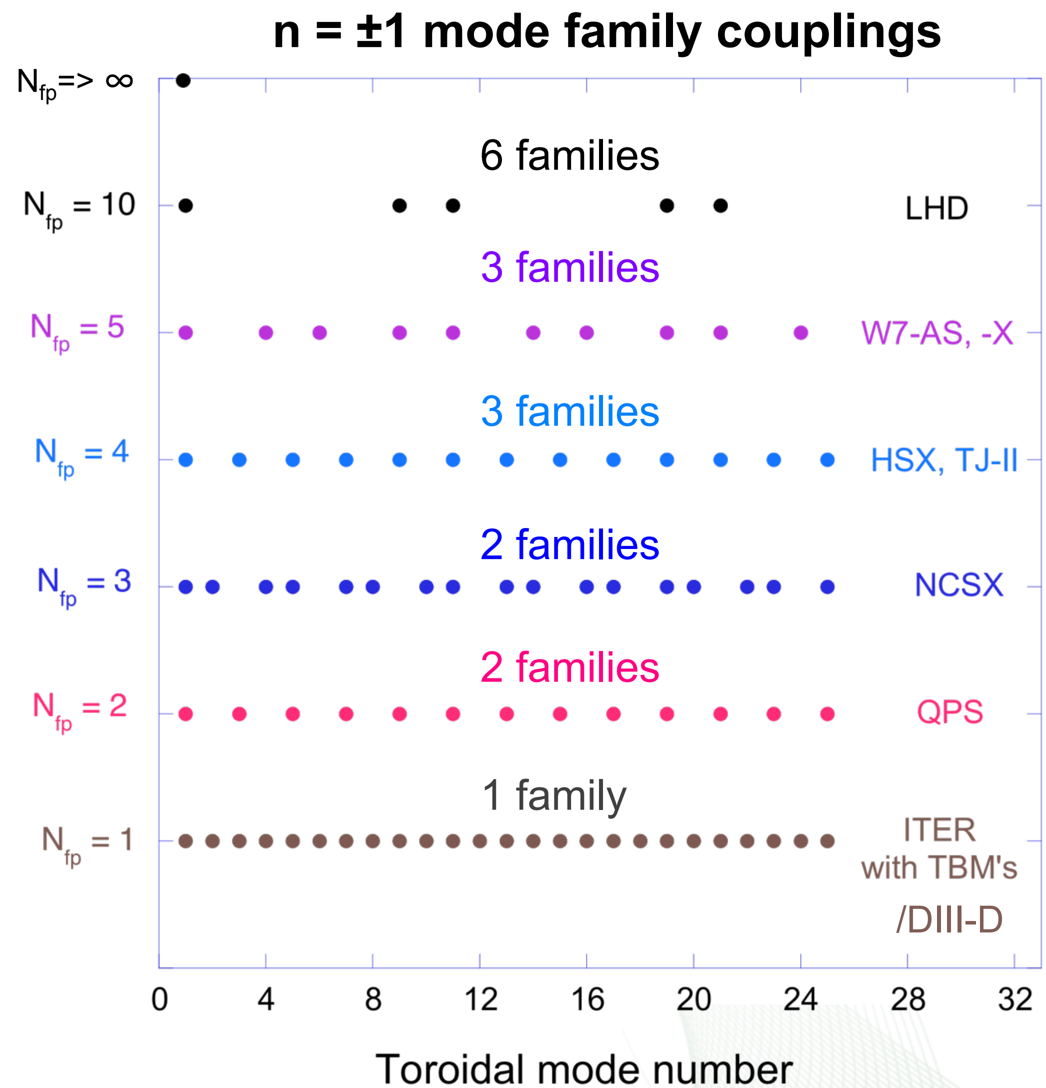
# Toroidal mode number (n) is not a good quantum number for 3D configurations

- Field period symmetry: $N_{fp}$ replicated elements
- Toroidal coupling => mode families, rather than single toroidal modes:

  $n' \pm n = kN_{fp}$, $k = 0, 1, 2, \ldots$

- Finite number of families: $1 + N_{fp}/2$ for even $N_{fp}$ and $(N_{fp}-1)/2 + 1$ for odd $N_{fp}$
- Computational difficulty
  - High $N_{fp}$ easiest
  - Low $N_{fp}$ hardest





# Gyrokinetic models for 3D configurations

- **PIC: domain divided up into cells**
  - grouped toroidally/radially for parallelization
- **Kinetic ions, fast ions: full GC orbits followed**
  - charges, currents allocated over local gyro radius template for field solve
- **Several options for electrons**
  - Adiabatic, Fluid/hybrid, Fully kinetic
- Electrostatic (ITG), electromagnetic (Alfvén instability)
- **Global vs. local**
  - GEM: flux tube approach generalized to flux surface – each field line different in 3D
  - Global: GTC, EUTERPE

# Global gyrokinetic model (GTC) for Alfvén and core turbulence in 3D systems

- Global, fully electromagnetic, nonlinear kinetic-MHD processes
- Gyrokinetic ions, fluid-kinetic hybrid electrons
- General 3D (VMEC) equilibria, ported to GPU and MIC
  - *n = 3* LHD Alfvén instability
  - *n = 10* LHD ITG instability

**Other GTC-related presentations:**

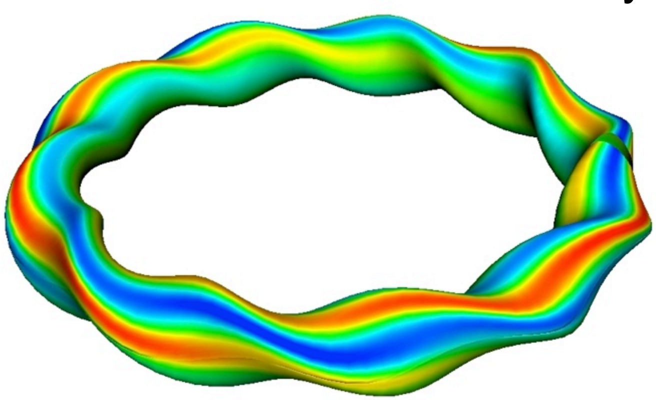
**Microturbulence:** JO3.00006, I. Holod; CP8.00006, Y. Xiao; CP8.00032, H. Xie; UP8.00006, D. Fulton

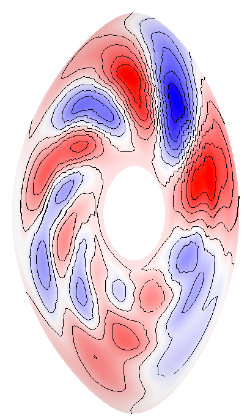
**EP:** NI1.00006, Z. Wang

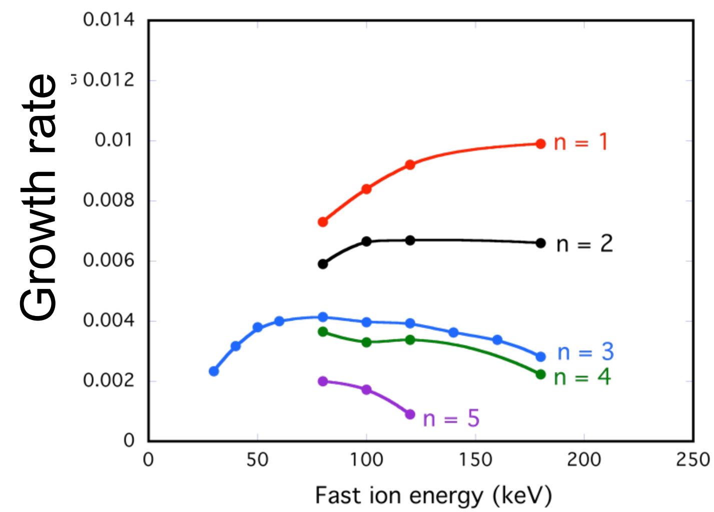
**MHD:** BP8.00046, D. Liu; TP8.00107, J. McClenaghan

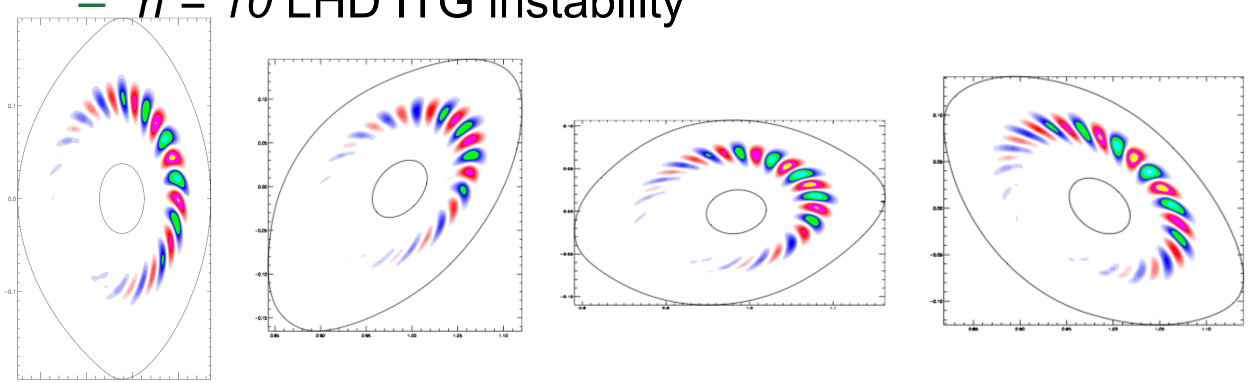
**RF:** NO3.00010, J. Bao; JP8.00080 X. Wei; TP8.00055, A. Kuley

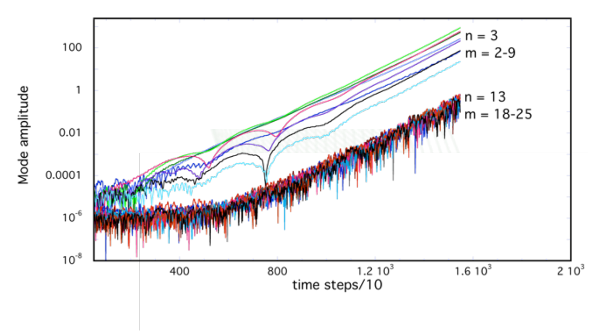
I. Holod, D. Spong, **Poster YP8.00011 APS-DPP 2014**



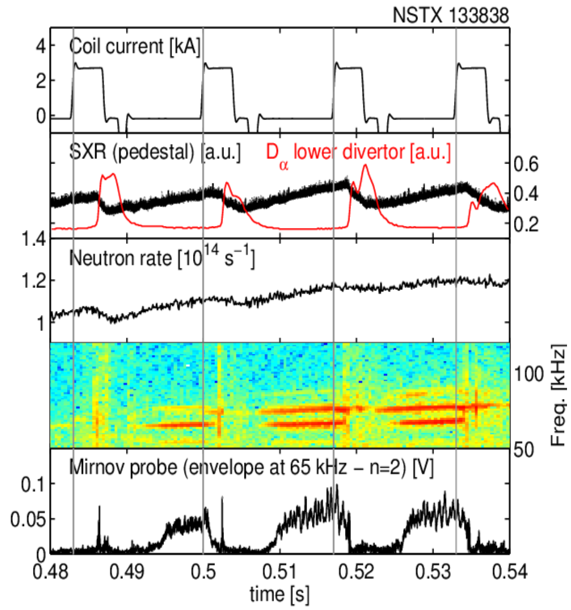




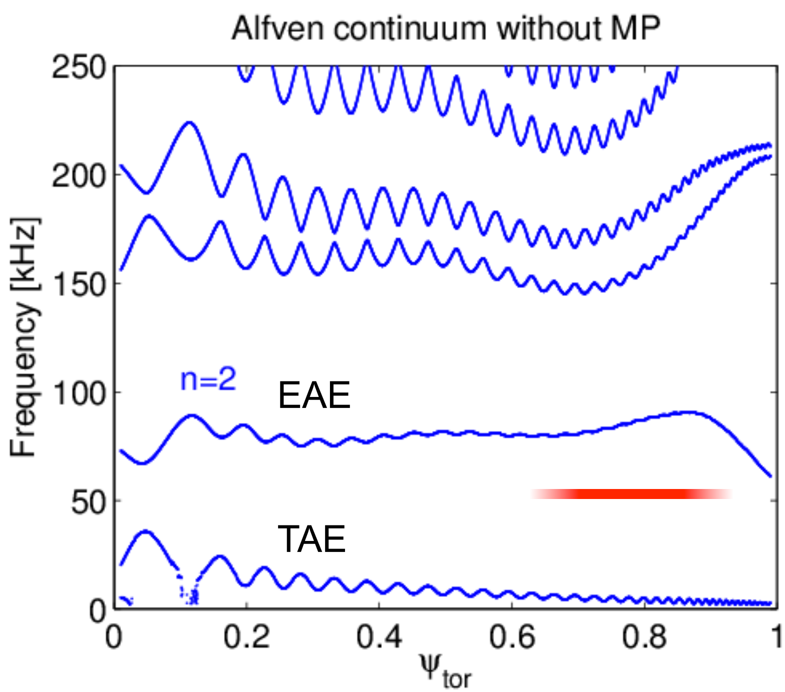



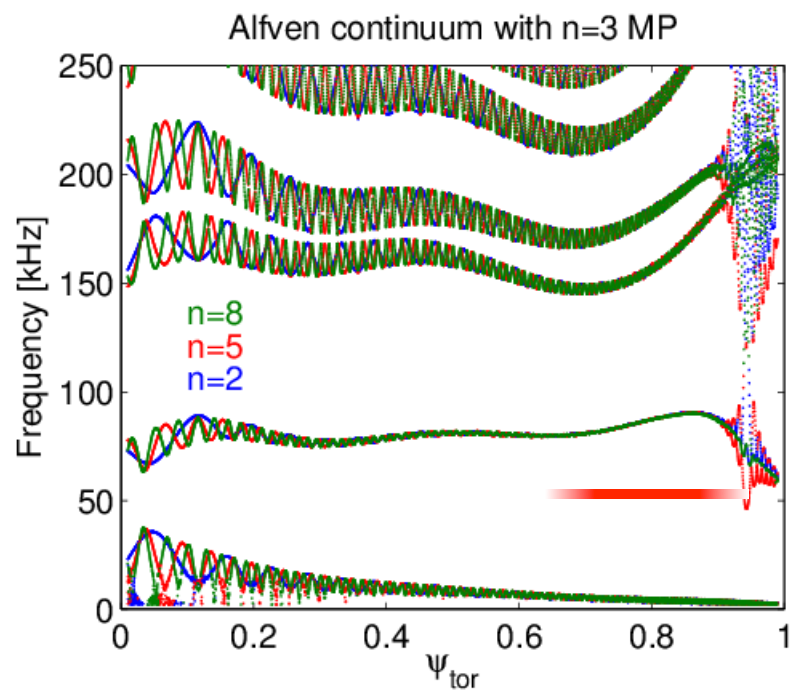


# NSTX Alfvén mode suppression correlated with 3D coils



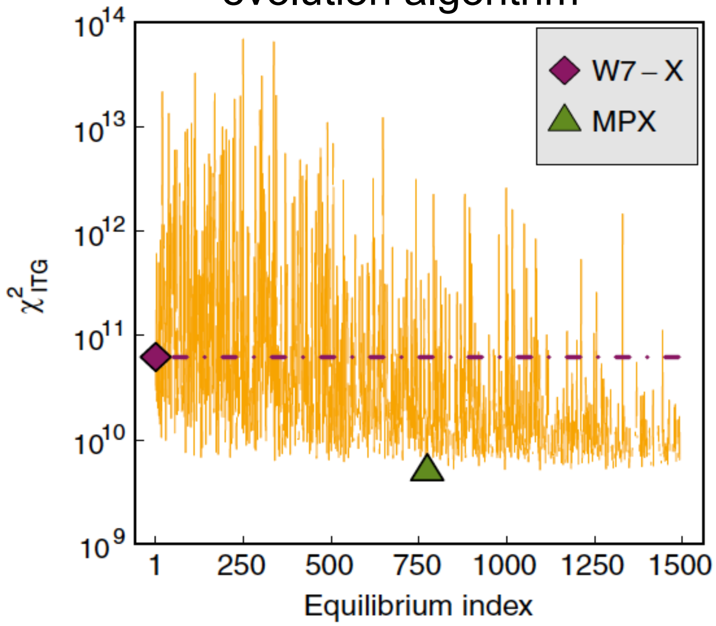

- A. Bortolon, PRL (2013), EPS (2014)
- ELMs triggered 2 ms after MP starts
- Two dominant TAEs observed
  - n=2, 65 kHz;  n=3, 75 kHz
- 3D Alfvén continuum (STELLGAP model) shows toroidal coupling effects near edge => increased damping for TAE



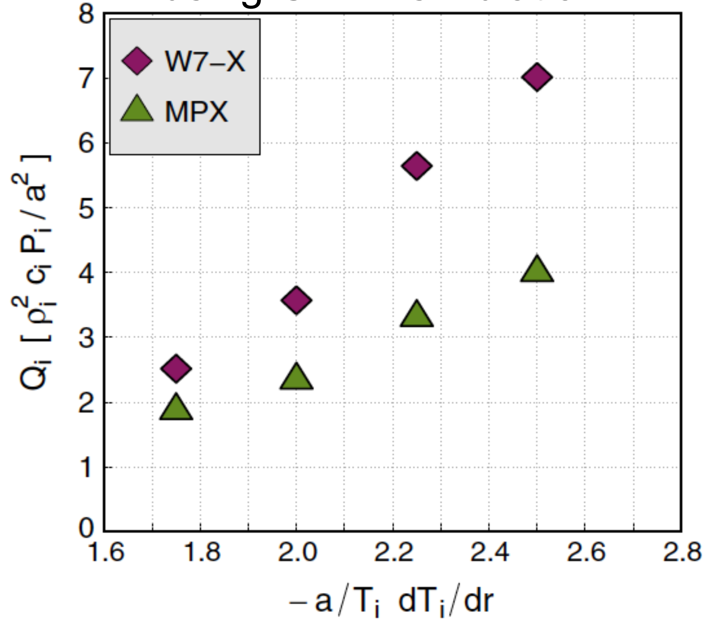



# Recent GENE optimizations show that ITG turbulence is sensitive to magnetic field geometry

STELLOPT with differential evolution algorithm

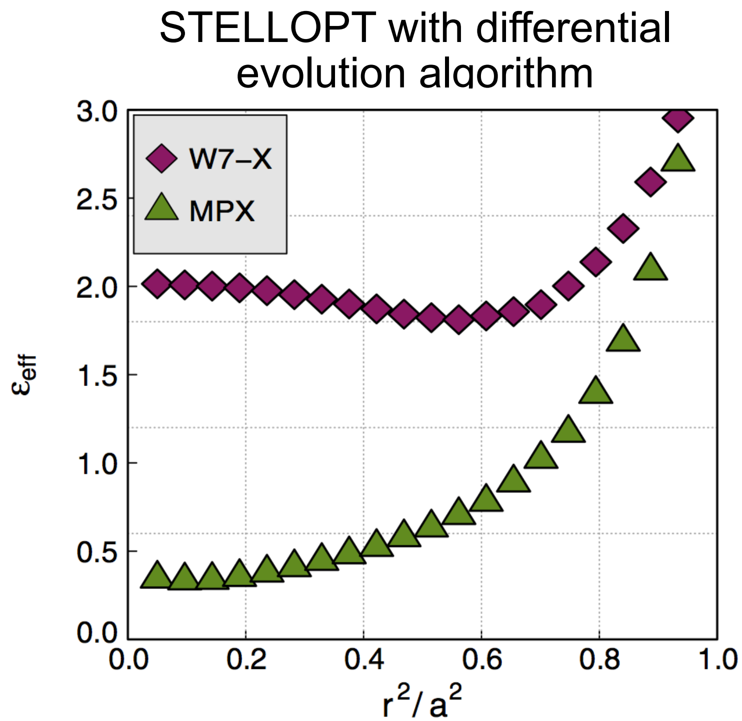


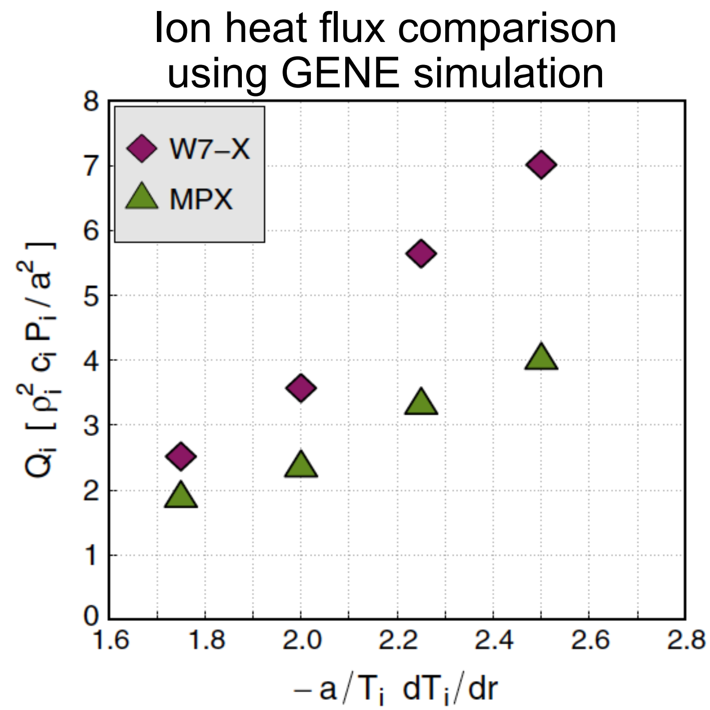

Ion heat flux comparison using GENE simulation




- GENE model: full flux surface simulation, gyrokinetic ions, Boltzmann electrons, electrostatic turbulence
- ITG optimization proxy: $\kappa_r^- \left(g^{rr}\right)^2$
- P. Xanthopoulos, H. Mynick, et al., Phys. Rev. Lett. **113**, Oct. 10, 2014
- H. Mynick, P. Xanthopoulos, et al., PPCF **56** (2014)

# Recent optimizations show that ITG turbulence sensitive to magnetic field geometry



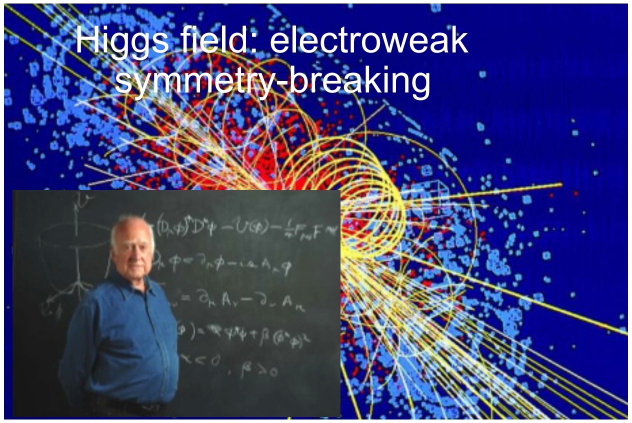




- GENE model: full flux surface simulation, gyrokinetic ions, Boltzmann electrons, electrostatic turbulence
- P. Xanthopoulos, H. Mynick, et al., Phys. Rev. Lett. **113**, Oct. 10, 2014
- J. Proll, et al., invited talk NO3-4, this meeting, TEM stability target: minimize particles with $\omega_{*e}\omega_{de} > 0$, applied to W7-X, HSX
- G. Weir, invited talk TI1.02, this meeting, GENE application to HSX

# Conclusions/Summary

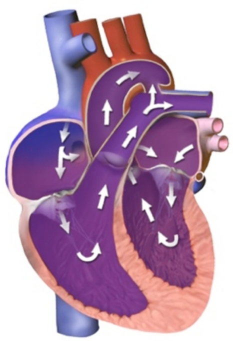
- **3D toroidal physics present in tokamaks/stellarators/ reversed field pinches**
  - Excellent opportunity for testing/validating theory with new models applicable to all systems
- **Increasing computational resources and new theoretical methods aid in the challenge of 3D**
- **3D design/optimization: multi-physics integration, deeper design space than 2D. Opportunities:**
  - Improved RFP confinement/sustainment
  - New directions in stellarator optimization (transport, microturbulence, MHD, energetic particle physics)
  - Tokamak 3D edge (better optimization for ELM/RWM suppression, divertor structure, control of detachment and scrape-off layer)

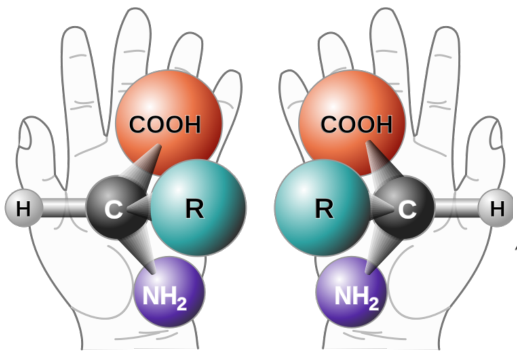
# We're not alone in exploring symmetry-breaking: prevalent in nature as well as human-made objects:


Higgs field: electroweak symmetry-breaking

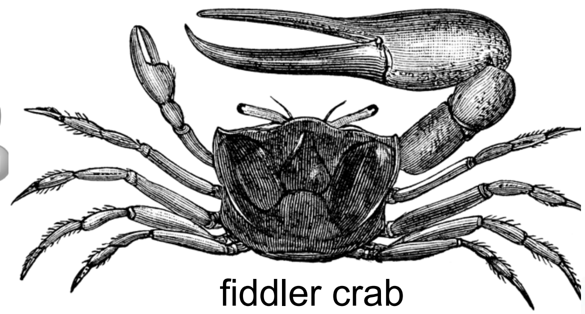


Frank Gehry architecture


Heart: left-right asymmetry

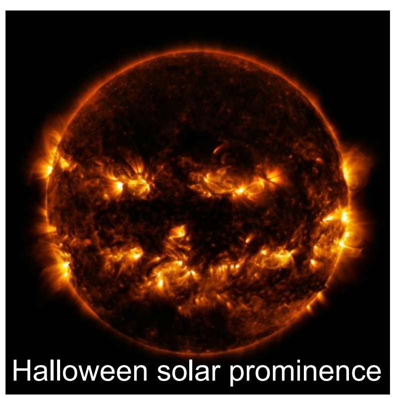


chiral amino acid molecules


fiddler crab


camshaft


Van Allen belts


Halloween solar prominence


DNA: right-handed helix only


Rutan asymmetric aircraft